Fundamental Data Structures

Contents

Introduction

Abstract data type

Not to be confused with Algebraic data type.

In computer science, an **abstract data type** (**ADT**) is a mathematical model for a certain class of data structures that have similar behavior; or for certain data types of one or more programming languages that have similar semantics. An abstract data type is defined indirectly, only by the operations that may be performed on it and by mathematical constraints on the effects (and possibly cost) of those operations.[1]

For example, an abstract stack could be defined by three operations: `push`, that inserts some data item onto the structure, `pop`, that extracts an item from it (with the constraint that each pop always returns the most recently pushed item that has not been popped yet), and `peek`, that allows data on top of the structure to be examined without removal. When analyzing the efficiency of algorithms that use stacks, one may also specify that all operations take the same time no matter how many items have been pushed into the stack, and that the stack uses a constant amount of storage for each element.

Abstract data types are purely theoretical entities, used (among other things) to simplify the description of abstract algorithms, to classify and evaluate data structures, and to formally describe the type systems of programming languages. However, an ADT may be implemented by specific data types or data structures, in many ways and in many programming languages; or described in a formal specification language. ADTs are often implemented as modules: the module's interface declares procedures that correspond to the ADT operations, sometimes with comments that describe the constraints. This information hiding strategy allows the implementation of the module to be changed without disturbing the client programs.

The term **abstract data type** can also be regarded as a generalised approach of a number of algebraic structures, such as lattices, groups, and rings.[2] This can be treated as part of the subject area of artificial intelligence. The notion of abstract data types is related to the concept of data abstraction, important in object-oriented programming and design by contract methodologies for software development Wikipedia:Citation needed.

Defining an abstract data type (*ADT*)

An abstract data type is defined as a mathematical model of the data objects that make up a data type as well as the functions that operate on these objects. There are no standard conventions for defining them. A broad division may be drawn between "imperative" and "functional" definition styles.

IB Computer Science Abstract Data Type

In the course an abstract data type refers to a generalised data structure that accepts data objects stored as a list with specific behaviours defined by the methods associated with the underlying nature of the list.

This is a very important BIG idea in computer science. It is based on the recognitation that often a group of data is simply a list in random or some specific order. A list of numbers can be either integer or real but as in arithmetic the numbers can represent any numeric quantity e.g. weight of a list of students in a classroom, in which case we would have a list of real numbers. Mathematical operations e.g. average, min, max are performed exactly the same way on any list of real numbers irrespective of the specific concrete nature of the list.

For example: a group of people queueing at the canteen can be represented as a list with certain characteristics or behaviours. People arrive and attach to the end of the queue, people get served and leave the queue from the head or start of the queue. Any simple queue can be described in exactly the same way. The same basic operations: add, remove, insert are always the same, it does not matter that the data object represent a person or something else. Adding the object to end of the queue is exactly the same operation for any set of data described as a queue.

An ADT has a generalised name e.g. Stack, Queue, BinaryTree etc. Each ADT accepts data objects that are represented as members of the underlying list e.g. an integer, a Person Object. Each ADT has a set of pre-defined methods (behaviours in OOPs terminology) that can be used to manipulate the members in the list - irrespective of what they actually in reality represent.

Other specific ADT material can be found in specific Wikipedia sections e.g. Stack, Queue, Array etc.

Imperative view

In the "imperative" view, which is closer to the philosophy of imperative programming languages, an abstract data structure is conceived as an entity that is *mutable* — meaning that it may be in different *states* at different times. Some operations may change the state of the ADT; therefore, the order in which operations are evaluated is important, and the same operation on the same entities may have different effects if executed at different times — just like the instructions of a computer, or the commands and procedures of an imperative language. To underscore this view, it is customary to say that the operations are *executed* or *applied*, rather than *evaluated*. The imperative style is often used when describing abstract algorithms. This is described by Donald E. Knuth and can be referenced from here The Art of Computer Programming.

Abstract variable

Imperative ADT definitions often depend on the concept of an *abstract variable*, which may be regarded as the simplest non-trivial ADT. An abstract variable *V* is a mutable entity that admits two operations:

- `store`(*V*,*x*) where *x* is a *value* of unspecified nature; and
- `fetch`(*V*), that yields a value;

with the constraint that

- `fetch`(*V*) always returns the value *x* used in the most recent `store`(*V*,*x*) operation on the same variable *V*.

As in so many programming languages, the operation `store`(*V*,*x*) is often written $V \leftarrow x$ (or some similar notation), and `fetch`(*V*) is implied whenever a variable *V* is used in a context where a value is required. Thus, for example, $V \leftarrow V + 1$ is commonly understood to be a shorthand for `store`(*V*,`fetch`(*V*) + 1).

In this definition, it is implicitly assumed that storing a value into a variable *U* has no effect on the state of a distinct variable *V*. To make this assumption explicit, one could add the constraint that

- if *U* and *V* are distinct variables, the sequence { `store`(*U*,*x*); `store`(*V*,*y*) } is equivalent to { `store`(*V*,*y*); `store`(*U*,*x*) }.

More generally, ADT definitions often assume that any operation that changes the state of one ADT instance has no effect on the state of any other instance (including other instances of the same ADT) — unless the ADT axioms imply that the two instances are connected (aliased) in that sense. For example, when extending the definition of abstract variable to include abstract records, the operation that selects a field from a record variable *R* must yield a variable *V* that is aliased to that part of *R*.

The definition of an abstract variable *V* may also restrict the stored values *x* to members of a specific set *X*, called the *range* or *type* of *V*. As in programming languages, such restrictions may simplify the description and analysis of algorithms, and improve their readability.

Note that this definition does not imply anything about the result of evaluating `fetch`(*V*) when *V* is *un-initialized*, that is, before performing any `store` operation on *V*. An algorithm that does so is usually considered invalid, because its effect is not defined. (However, there are some important algorithms whose efficiency strongly depends on the assumption that such a `fetch` is legal, and returns some arbitrary value in the variable's range.Wikipedia:Citation needed)

Instance creation

Some algorithms need to create new instances of some ADT (such as new variables, or new stacks). To describe such algorithms, one usually includes in the ADT definition a `create`() operation that yields an instance of the ADT, usually with axioms equivalent to

- the result of `create`() is distinct from any instance *S* in use by the algorithm.

This axiom may be strengthened to exclude also partial aliasing with other instances. On the other hand, this axiom still allows implementations of `create`() to yield a previously created instance that has become inaccessible to the program.

Preconditions, postconditions, and invariants

In imperative-style definitions, the axioms are often expressed by *preconditions*, that specify when an operation may be executed; *postconditions*, that relate the states of the ADT before and after the execution of each operation; and *invariants*, that specify properties of the ADT that are *not* changed by the operations.

Example: abstract stack (imperative)

As another example, an imperative definition of an abstract stack could specify that the state of a stack *S* can be modified only by the operations

- `push`(*S*,*x*), where *x* is some *value* of unspecified nature; and
- `pop`(*S*), that yields a value as a result;

with the constraint that

- For any value *x* and any abstract variable *V*, the sequence of operations { `push`(*S*,*x*); $V \leftarrow$ `pop`(*S*) } is equivalent to { $V \leftarrow x$ };

Since the assignment { $V \leftarrow x$ }, by definition, cannot change the state of *S*, this condition implies that { $V \leftarrow$ `pop`(*S*) } restores *S* to the state it had before the { `push`(*S*,*x*) }. From this condition and from the properties of abstract variables, it follows, for example, that the sequence

{ `push`(*S*,*x*); `push`(*S*,*y*); $U \leftarrow$ `pop`(*S*); `push`(*S*,*z*); $V \leftarrow$ `pop`(*S*); $W \leftarrow$ `pop`(*S*); }

where *x*,*y*, and *z* are any values, and *U*, *V*, *W* are pairwise distinct variables, is equivalent to

{ $U \leftarrow y$; $V \leftarrow z$; $W \leftarrow x$ }

Here it is implicitly assumed that operations on a stack instance do not modify the state of any other ADT instance, including other stacks; that is,

- For any values *x*,*y*, and any distinct stacks *S* and *T*, the sequence { `push`(*S*,*x*); `push`(*T*,*y*) } is equivalent to { `push`(*T*,*y*); `push`(*S*,*x*) }.

A stack ADT definition usually includes also a Boolean-valued function `empty`(*S*) and a `create`() operation that returns a stack instance, with axioms equivalent to

- `create`() ≠ *S* for any stack *S* (a newly created stack is distinct from all previous stacks)
- `empty`(`create`()) (a newly created stack is empty)
- `not empty`(`push`(*S*,*x*)) (pushing something into a stack makes it non-empty)

Single-instance style

Sometimes an ADT is defined as if only one instance of it existed during the execution of the algorithm, and all operations were applied to that instance, which is not explicitly notated. For example, the abstract stack above could have been defined with operations `push`(*x*) and `pop`(), that operate on "the" only existing stack. ADT definitions in this style can be easily rewritten to admit multiple coexisting instances of the ADT, by adding an explicit instance parameter (like *S* in the previous example) to every operation that uses or modifies the implicit instance.

On the other hand, some ADTs cannot be meaningfully defined without assuming multiple instances. This is the case when a single operation takes two distinct instances of the ADT as parameters. For an example, consider augmenting the definition of the stack ADT with an operation `compare`(*S*,*T*) that checks whether the stacks *S* and *T* contain the same items in the same order.

Functional ADT definitions

Another way to define an ADT, closer to the spirit of functional programming, is to consider each state of the structure as a separate entity. In this view, any operation that modifies the ADT is modeled as a mathematical function that takes the old state as an argument, and returns the new state as part of the result. Unlike the "imperative" operations, these functions have no side effects. Therefore, the order in which they are evaluated is immaterial, and the same operation applied to the same arguments (including the same input states) will always return the same results (and output states).

In the functional view, in particular, there is no way (or need) to define an "abstract variable" with the semantics of imperative variables (namely, with `fetch` and `store` operations). Instead of storing values into variables, one passes them as arguments to functions.

Example: abstract stack (functional)

For example, a complete functional-style definition of a stack ADT could use the three operations:

- `push`: takes a stack state and an arbitrary value, returns a stack state;
- `top`: takes a stack state, returns a value;
- `pop`: takes a stack state, returns a stack state;

In a functional-style definition there is no need for a `create` operation. Indeed, there is no notion of "stack instance". The stack states can be thought of as being potential states of a single stack structure, and two stack states that contain the same values in the same order are considered to be identical states. This view actually mirrors the behavior of some concrete implementations, such as linked lists with hash cons.

Instead of `create`(), a functional definition of a stack ADT may assume the existence of a special stack state, the *empty stack*, designated by a special symbol like Λ or "()"; or define a `bottom`() operation that takes no arguments and returns this special stack state. Note that the axioms imply that

- `push`(Λ,*x*) ≠ Λ

In a functional definition of a stack one does not need an `empty` predicate: instead, one can test whether a stack is empty by testing whether it is equal to Λ.

Note that these axioms do not define the effect of `top`(*s*) or `pop`(*s*), unless *s* is a stack state returned by a `push`. Since `push` leaves the stack non-empty, those two operations are undefined (hence invalid) when *s* = Λ. On the other hand, the axioms (and the lack of side effects) imply that `push`(*s*,*x*) = `push`(*t*,*y*) if and only if *x* = *y* and *s* = *t*.

As in some other branches of mathematics, it is customary to assume also that the stack states are only those whose existence can be proved from the axioms in a finite number of steps. In the stack ADT example above, this rule means that every stack is a *finite* sequence of values, that becomes the empty stack (Λ) after a finite number of `pop`s. By themselves, the axioms above do not exclude the existence of infinite stacks (that can be `pop`ed forever, each time yielding a different state) or circular stacks (that return to the same state after a finite number of `pop`s). In particular, they do not exclude states *s* such that `pop`(*s*) = *s* or `push`(*s*,*x*) = *s* for some *x*. However, since one cannot obtain such stack states with the given operations, they are assumed "not to exist".

Advantages of abstract data typing

- Encapsulation

Abstraction provides a promise that any implementation of the ADT has certain properties and abilities; knowing these is all that is required to make use of an ADT object. The user does not need any technical knowledge of how the implementation works to use the ADT. In this way, the implementation may be complex but will be encapsulated in a simple interface when it is actually used.

- Localization of change

Code that uses an ADT object will not need to be edited if the implementation of the ADT is changed. Since any changes to the implementation must still comply with the interface, and since code using an ADT may only refer to properties and abilities specified in the interface, changes may be made to the implementation without requiring any changes in code where the ADT is used.

- Flexibility

Different implementations of an ADT, having all the same properties and abilities, are equivalent and may be used somewhat interchangeably in code that uses the ADT. This gives a great deal of flexibility when using ADT objects in different situations. For example, different implementations of an ADT may be more efficient in different situations; it is possible to use each in the situation where they are preferable, thus increasing overall efficiency.

Typical operations

Some operations that are often specified for ADTs (possibly under other names) are

- `compare`(*s*,*t*), that tests whether two structures are equivalent in some sense;
- `hash`(*s*), that computes some standard hash function from the instance's state;
- `print`(*s*) or `show`(*s*), that produces a human-readable representation of the structure's state.

In imperative-style ADT definitions, one often finds also

- `create`(), that yields a new instance of the ADT;
- `initialize`(*s*), that prepares a newly created instance *s* for further operations, or resets it to some "initial state";
- `copy`(*s*,*t*), that puts instance *s* in a state equivalent to that of *t*;
- `clone`(*t*), that performs *s* ← `create`(), `copy`(*s*,*t*), and returns *s*;
- `free`(*s*) or `destroy`(*s*), that reclaims the memory and other resources used by *s*;

The `free` operation is not normally relevant or meaningful, since ADTs are theoretical entities that do not "use memory". However, it may be necessary when one needs to analyze the storage used by an algorithm that uses the ADT. In that case one needs additional axioms that specify how much memory each ADT instance uses, as a function of its state, and how much of it is returned to the pool by `free`.

Examples

Some common ADTs, which have proved useful in a great variety of applications, are

- Container
- Deque
- List
- Map
- Multimap
- Multiset
- Priority queue
- Queue
- Set
- Stack
- Tree
- Graph

Each of these ADTs may be defined in many ways and variants, not necessarily equivalent. For example, a stack ADT may or may not have a `count` operation that tells how many items have been pushed and not yet popped. This choice makes a difference not only for its clients but also for the implementation.

Implementation

Implementing an ADT means providing one procedure or function for each abstract operation. The ADT instances are represented by some concrete data structure that is manipulated by those procedures, according to the ADT's specifications.

Usually there are many ways to implement the same ADT, using several different concrete data structures. Thus, for example, an abstract stack can be implemented by a linked list or by an array.

An ADT implementation is often packaged as one or more modules, whose interface contains only the signature (number and types of the parameters and results) of the operations. The implementation of the module — namely, the bodies of the procedures and the concrete data structure used — can then be hidden from most clients of the module. This makes it possible to change the implementation without affecting the clients.

When implementing an ADT, each instance (in imperative-style definitions) or each state (in functional-style definitions) is usually represented by a handle of some sort.[3]

Modern object-oriented languages, such as C++ and Java, support a form of abstract data types. When a class is used as a type, it is an abstract type that refers to a hidden representation. In this model an ADT is typically implemented as a class, and each instance of the ADT is usually an object of that class. The module's interface typically declares the constructors as ordinary procedures, and most of the other ADT operations as methods of that class. However, such an approach does not easily encapsulate multiple representational variants found in an ADT. It also can undermine the extensibility of object-oriented programs. In a pure object-oriented program that uses interfaces as types, types refer to behaviors not representations.

Example: implementation of the stack ADT

As an example, here is an implementation of the stack ADT above in the C programming language.

Imperative-style interface

An imperative-style interface might be:

```
typedef struct stack_Rep stack_Rep;          /* Type: instance
representation (an opaque record). */
typedef stack_Rep *stack_T;                  /* Type: handle to a stack
instance (an opaque pointer). */
typedef void *stack_Item;                    /* Type: value that can be
stored in stack (arbitrary address). */

stack_T stack_create(void);                  /* Create new stack
instance, initially empty. */
void stack_push(stack_T s, stack_Item e);    /* Add an item at the top of
 the stack. */
stack_Item stack_pop(stack_T s);             /* Remove the top item from
the stack and return it . */
int stack_empty(stack_T ts);                 /* Check whether stack is
empty. */
```

This implementation could be used in the following manner:

```
#include <stack.h>              /* Include the stack interface. */
stack_T t = stack_create();     /* Create a stack instance. */
int foo = 17;                   /* An arbitrary datum. */
stack_push(t, &foo);            /* Push the address of 'foo' onto the
stack. */
…
void *e = stack_pop(t);         /* Get the top item and delete it from
the stack. */
if (stack_empty(t)) { … }       /* Do something if stack is empty. */
…
```

This interface can be implemented in many ways. The implementation may be arbitrarily inefficient, since the formal definition of the ADT, above, does not specify how much space the stack may use, nor how long each operation should take. It also does not specify whether the stack state *t* continues to exist after a call $s \leftarrow \texttt{pop}(t)$.

In practice the formal definition should specify that the space is proportional to the number of items pushed and not yet popped; and that every one of the operations above must finish in a constant amount of time, independently of that number. To comply with these additional specifications, the implementation could use a linked list, or an array (with dynamic resizing) together with two integers (an item count and the array size)

Functional-style interface

Functional-style ADT definitions are more appropriate for functional programming languages, and vice-versa. However, one can provide a functional style interface even in an imperative language like C. For example:

```
typedef struct stack_Rep stack_Rep;          /* Type: stack state
representation (an opaque record). */
typedef stack_Rep *stack_T;                  /* Type: handle to a stack
 state (an opaque pointer). */
typedef void *stack_Item;                    /* Type: item (arbitrary
address). */

stack_T stack_empty(void);                   /* Returns the empty stack
 state. */
stack_T stack_push(stack_T s, stack_Item x); /* Adds x at the top of s,
 returns the resulting state. */
stack_Item stack_top(stack_T s);             /* Returns the item
currently at the top of s. */
stack_T stack_pop(stack_T s);                /* Remove the top item
from s, returns the resulting state. */
```

The main problem is that C lacks garbage collection, and this makes this style of programming impractical; moreover, memory allocation routines in C are slower than allocation in a typical garbage collector, thus the performance impact of so many allocations is even greater.

ADT libraries

Many modern programming languages, such as C++ and Java, come with standard libraries that implement several common ADTs, such as those listed above.

Built-in abstract data types

The specification of some programming languages is intentionally vague about the representation of certain built-in data types, defining only the operations that can be done on them. Therefore, those types can be viewed as "built-in ADTs". Examples are the arrays in many scripting languages, such as Awk, Lua, and Perl, which can be regarded as an implementation of the Map or Table ADT.

References

[1] Barbara Liskov, Programming with Abstract Data Types, in Proceedings of the ACM SIGPLAN Symposium on Very High Level Languages, pp. 50--59, 1974, Santa Monica, California
[2] , Chapter 7,section 40.
[3] , definition 4.4.

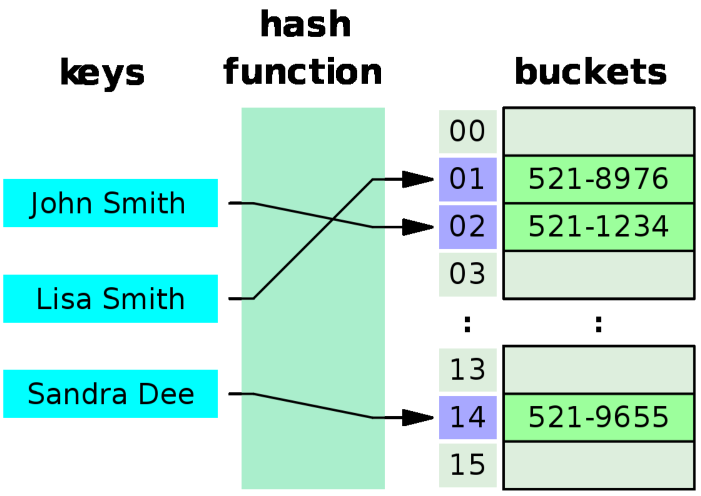
Further

• Mitchell, John C.; Plotkin, Gordon (July 1988). "Abstract Types Have Existential Type" (http://theory.stanford.edu/~jcm/papers/mitch-plotkin-88.pdf). *ACM Transactions on Programming Languages and Systems* **10** (3).

External links

- Abstract data type (http://www.nist.gov/dads/HTML/abstractDataType.html) in NIST Dictionary of Algorithms and Data Structures
- Walls and Mirrors, the classic textbook

Data structure



A hash table

In computer science, a **data structure** is a particular way of organizing data in a computer so that it can be used efficiently.[1][2]

Different kinds of data structures are suited to different kinds of applications, and some are highly specialized to specific tasks. For example, B-trees are particularly well-suited for implementation of databases, while compiler implementations usually use hash tables to look up identifiers.

Data structures provide a means to manage large amounts of data efficiently, such as large databases and internet indexing services. Usually, efficient data structures are a key in designing efficient algorithms. Some formal design methods and programming languages emphasize data structures, rather than algorithms, as the key organizing factor in software design. Storing and retrieving can be carried out on data stored in both main memory and in secondary memory.

Overview

Data structures are generally based on the ability of a computer to fetch and store data at any place in its memory, specified by an address – a bit string that can be itself stored in memory and manipulated by the program. Thus, the array and record data structures are based on computing the addresses of data items with arithmetic operations; while the linked data structures are based on storing addresses of data items within the structure itself. Many data structures use both principles, sometimes combined in non-trivial ways (as in XOR linking).

The implementation of a data structure usually requires writing a set of procedures that create and manipulate instances of that structure. The efficiency of a data structure cannot be analyzed separately from those operations. This observation motivates the theoretical concept of an abstract data type, a data structure that is defined indirectly by the operations that may be performed on it, and the mathematical properties of those operations (including their space and time cost).

Examples

Main article: List of data structures

There are numerous types of data structures:

- An *array* is a number of elements in a specific order. They are accessed using an integer to specify which element is required (although the elements may be of almost any type). Typical implementations allocate contiguous memory words for the elements of arrays (but this is not always a necessity). Arrays may be fixed-length or expandable.
- *Records* (also called *tuples* or *structs*) are among the simplest data structures. A record is a value that contains other values, typically in fixed number and sequence and typically indexed by names. The elements of records are usually called *fields* or *members*.
- An *associative array* (also called a *dictionary* or *map*) is a more flexible variation on a record, in which name-value pairs can be added and deleted freely. A hash table is a common implementation of an associative array.
- A *union* type specifies which of a number of permitted primitive types may be stored in its instances, e.g. "float or long integer". Contrast with a record, which could be defined to contain a float *and* an integer; whereas, in a union, there is only one value at a time. Enough space is allocated to contain the widest member datatype.
- A *tagged union* (also called a *variant*, *variant record*, *discriminated union*, or *disjoint union*) contains an additional field indicating its current type, for enhanced type safety.
- A *set* is an abstract data structure that can store specific values, without any particular order, and with no repeated values. Values themselves are not retrieved from sets, rather one tests a value for membership to obtain a boolean "in" or "not in".
- *Graphs* and *trees* are linked abstract data structures composed of *nodes*. Each node contains a value and also one or more pointers to other nodes. Graphs can be used to represent networks, while variants of trees can be used for sorting and searching, having their nodes arranged in some relative order based on their values.
- An *object* contains data fields, like a record, as well as various methods. In the context of object-oriented programming, records are known as plain old data structures to distinguish them from objects.

Language support

Most assembly languages and some low-level languages, such as BCPL (Basic Combined Programming Language), lack support for data structures. On the other hand, many high-level programming languages and some higher-level assembly languages, such as MASM, have special syntax or other built-in support for certain data structures, such as records and arrays. For example, the C and Pascal languages support structs and records, respectively, in addition to vectors (one-dimensional arrays) and multi-dimensional arrays.

Most programming languages feature some sort of library mechanism that allows data structure implementations to be reused by different programs. Modern languages usually come with standard libraries that implement the most common data structures. Examples are the C++ Standard Template Library, the Java Collections Framework, and Microsoft's .NET Framework.

Modern languages also generally support modular programming, the separation between the interface of a library module and its implementation. Some provide opaque data types that allow clients to hide implementation details. Object-oriented programming languages, such as C++, Java and Smalltalk may use classes for this purpose.

Many known data structures have concurrent versions that allow multiple computing threads to access the data structure simultaneously.

References

[1] Paul E. Black (ed.), entry for *data structure* in *Dictionary of Algorithms and Data Structures. U.S. National Institute of Standards and Technology. 15 December 2004. Online version* (http://xlinux.nist.gov/dads/HTML/datastructur.html) *Accessed May 21, 2009.*

[2] Entry *data structure* in the Encyclopædia Britannica (2009) Online entry (http://www.britannica.com/EBchecked/topic/152190/data-structure) accessed on May 21, 2009.

Further reading

- Peter Brass, *Advanced Data Structures*, Cambridge University Press, 2014
- Donald Knuth, *The Art of Computer Programming*, vol. 1. Addison-Wesley, 3rd edition, 1997.
- Dinesh Mehta and Sartaj Sahni *Handbook of Data Structures and Applications*, Chapman and Hall/CRC Press, 2007.
- Niklaus Wirth, *Algorithms and Data Structures*, Prentice Hall, 1985.
- Diane Zak, Introduction to programming with c++, copyright 2011 Cengage Learning Asia Pte Ltd

External links

- course on data structures (http://scanftree.com/Data_Structure/)
- Data structures Programs Examples in c,java (http://scanftree.com/programs/operation/data-structure/)
- UC Berkeley video course on data structures (http://academicearth.org/computer-science/)
- Descriptions (http://nist.gov/dads/) from the Dictionary of Algorithms and Data Structures
- Data structures course (http://www.cs.auckland.ac.nz/software/AlgAnim/ds_ToC.html)
- An Examination of Data Structures from .NET perspective (http://msdn.microsoft.com/en-us/library/aa289148(VS.71).aspx)
- Schaffer, C. *Data Structures and Algorithm Analysis* (http://people.cs.vt.edu/~shaffer/Book/C++3e20110915.pdf)

Analysis of algorithms

In computer science, the **analysis of algorithms** is the determination of the amount of resources (such as time and storage) necessary to execute them. Most algorithms are designed to work with inputs of arbitrary length. Usually, the efficiency or running time of an algorithm is stated as a function relating the input length to the number of steps (time complexity) or storage locations (space complexity).

Algorithm analysis is an important part of a broader computational complexity theory, which provides theoretical estimates for the resources needed by any algorithm which solves a given computational problem. These estimates provide an insight into reasonable directions of search for efficient algorithms.

In theoretical analysis of algorithms it is common to estimate their complexity in the asymptotic sense, i.e., to estimate the complexity function for arbitrarily large input. Big O notation, Big-omega notation and Big-theta notation are used to this end. For instance, binary search is said to run in a number of steps proportional to the logarithm of the length of the list being searched, or in O(log(n)), colloquially "in logarithmic time". Usually asymptotic estimates are used because different implementations of the same algorithm may differ in efficiency. However the efficiencies of any two "reasonable" implementations of a given algorithm are related by a constant multiplicative factor called a *hidden constant*.

Exact (not asymptotic) measures of efficiency can sometimes be computed but they usually require certain assumptions concerning the particular implementation of the algorithm, called model of computation. A model of computation may be defined in terms of an abstract computer, e.g., Turing machine, and/or by postulating that certain operations are executed in unit time. For example, if the sorted list to which we apply binary search has n elements, and we can guarantee that each lookup of an element in the list can be done in unit time, then at most $\log_2 n + 1$ time units are needed to return an answer.

Cost models

Time efficiency estimates depend on what we define to be a step. For the analysis to correspond usefully to the actual execution time, the time required to perform a step must be guaranteed to be bounded above by a constant. One must be careful here; for instance, some analyses count an addition of two numbers as one step. This assumption may not be warranted in certain contexts. For example, if the numbers involved in a computation may be arbitrarily large, the time required by a single addition can no longer be assumed to be constant.

Two cost models are generally used:[1]

- the **uniform cost model**, also called **uniform-cost measurement** (and similar variations), assigns a constant cost to every machine operation, regardless of the size of the numbers involved
- the **logarithmic cost model**, also called **logarithmic-cost measurement** (and variations thereof), assigns a cost to every machine operation proportional to the number of bits involved

The latter is more cumbersome to use, so it's only employed when necessary, for example in the analysis of arbitrary-precision arithmetic algorithms, like those used in cryptography.

A key point which is often overlooked is that published lower bounds for problems are often given for a model of computation that is more restricted than the set of operations that you could use in practice and therefore there are algorithms that are faster than what would naively be thought possible.[2]

Run-time analysis

Run-time analysis is a theoretical classification that estimates and anticipates the increase in *running time* (or run-time) of an algorithm as its *input size* (usually denoted as *n*) increases. Run-time efficiency is a topic of great interest in computer science: A program can take seconds, hours or even years to finish executing, depending on which algorithm it implements (see also performance analysis, which is the analysis of an algorithm's run-time in practice).

Shortcomings of empirical metrics

Since algorithms are platform-independent (i.e. a given algorithm can be implemented in an arbitrary programming language on an arbitrary computer running an arbitrary operating system), there are significant drawbacks to using an empirical approach to gauge the comparative performance of a given set of algorithms.

Take as an example a program that looks up a specific entry in a sorted list of size *n*. Suppose this program were implemented on Computer A, a state-of-the-art machine, using a linear search algorithm, and on Computer B, a much slower machine, using a binary search algorithm. Benchmark testing on the two computers running their respective programs might look something like the following:

***n* (list size)**	**Computer A run-time (in nanoseconds)**	**Computer B run-time (in nanoseconds)**
15	7	100,000
65	32	150,000
250	125	200,000
1,000	500	250,000

Based on these metrics, it would be easy to jump to the conclusion that *Computer A* is running an algorithm that is far superior in efficiency to that of *Computer B*. However, if the size of the input-list is increased to a sufficient number, that conclusion is dramatically demonstrated to be in error:

***n* (list size)**	**Computer A run-time (in nanoseconds)**	**Computer B run-time (in nanoseconds)**
15	7	100,000
65	32	150,000
250	125	200,000
1,000	500	250,000
...	...	...
1,000,000	500,000	500,000
4,000,000	2,000,000	550,000
16,000,000	8,000,000	600,000
...	...	...
$63{,}072 \times 10^{12}$	$31{,}536 \times 10^{12}$ ns, or 1 year	1,375,000 ns, or 1.375 milliseconds

Computer A, running the linear search program, exhibits a linear growth rate. The program's run-time is directly proportional to its input size. Doubling the input size doubles the run time, quadrupling the input size quadruples the run-time, and so forth. On the other hand, Computer B, running the binary search program, exhibits a logarithmic growth rate. Doubling the input size only increases the run time by a constant amount (in this example, 50,000 ns). Even though Computer A is ostensibly a faster machine, Computer B will inevitably surpass Computer A in run-time because it's running an algorithm with a much slower growth rate.

Orders of growth

Main article: Big O notation

Informally, an algorithm can be said to exhibit a growth rate on the order of a mathematical function if beyond a certain input size *n*, the function *f(n)* times a positive constant provides an upper bound or limit for the run-time of that algorithm. In other words, for a given input size *n* greater than some n_0 and a constant *c*, the running time of that algorithm will never be larger than $c \times f(n)$. This concept is frequently expressed using Big O notation. For example, since the run-time of insertion sort grows quadratically as its input size increases, insertion sort can be said to be of order $O(n^2)$.

Big O notation is a convenient way to express the worst-case scenario for a given algorithm, although it can also be used to express the average-case — for example, the worst-case scenario for quicksort is $O(n^2)$, but the average-case run-time is $O(n \log n)$.[3]

Empirical orders of growth

Assuming the execution time follows power rule, t □ $k\, n^a$, the coefficient *a* can be found [4] by taking empirical measurements of run time $\{t1, t2\}$ at some problem-size points $\{n1, n2\}$, and calculating $t_2/t_1 = (n_2/n_1)^a$ so that $a = \log(t_2/t_1)/\log(n_2/n_1)$. If the order of growth indeed follows the power rule, the empirical value of *a* will stay constant at different ranges, and if not, it will change - but still could serve for comparison of any two given algorithms as to their *empirical local orders of growth* behaviour. Applied to the above table:

n **(list size)**	**Computer A run-time (in nanoseconds)**	**Local order of growth (n^_)**	**Computer B run-time (in nanoseconds)**	**Local order of growth (n^_)**
15	7		100,000	
65	32	1.04	150,000	0.28
250	125	1.01	200,000	0.21
1,000	500	1.00	250,000	0.16
...	...		...	
1,000,000	500,000	1.00	500,000	0.10
4,000,000	2,000,000	1.00	550,000	0.07
16,000,000	8,000,000	1.00	600,000	0.06
...	...		...	

It is clearly seen that the first algorithm exhibits a linear order of growth indeed following the power rule. The empirical values for the second one are diminishing rapidly, suggesting it follows another rule of growth and in any case has much lower local orders of growth (and improving further still), empirically, than the first one.

Evaluating run-time complexity

The run-time complexity for the worst-case scenario of a given algorithm can sometimes be evaluated by examining the structure of the algorithm and making some simplifying assumptions. Consider the following pseudocode:

```
1    get a positive integer from input
2    if n > 10
3        print "This might take a while..."
4    for i = 1 to n
5        for j = 1 to i
6            print i * j
7    print "Done!"
```

A given computer will take a discrete amount of time to execute each of the instructions involved with carrying out this algorithm. The specific amount of time to carry out a given instruction will vary depending on which instruction is being executed and which computer is executing it, but on a conventional computer, this amount will be deterministic.[5] Say that the actions carried out in step 1 are considered to consume time T_1, step 2 uses time T_2, and so forth.

In the algorithm above, steps 1, 2 and 7 will only be run once. For a worst-case evaluation, it should be assumed that step 3 will be run as well. Thus the total amount of time to run steps 1-3 and step 7 is:

$$T_1 + T_2 + T_3 + T_7.$$

The loops in steps 4, 5 and 6 are trickier to evaluate. The outer loop test in step 4 will execute (n + 1) times (note that an extra step is required to terminate the for loop, hence n + 1 and not n executions), which will consume T_4(n + 1) time. The inner loop, on the other hand, is governed by the value of i, which iterates from 1 to i. On the first pass through the outer loop, j iterates from 1 to 1: The inner loop makes one pass, so running the inner loop body (step 6) consumes T_6 time, and the inner loop test (step 5) consumes $2T_5$ time. During the next pass through the outer loop, j iterates from 1 to 2: the inner loop makes two passes, so running the inner loop body (step 6) consumes $2T_6$ time, and the inner loop test (step 5) consumes $3T_5$ time.

Altogether, the total time required to run the inner loop body can be expressed as an arithmetic progression:

$$T_6 + 2T_6 + 3T_6 + \cdots + (n-1)T_6 + nT_6$$

which can be factored[6] as

$$T_6\left[1 + 2 + 3 + \cdots + (n-1) + n\right] = T_6\left[\frac{1}{2}(n^2 + n)\right]$$

The total time required to run the outer loop test can be evaluated similarly:

$$\begin{aligned} &2T_5 + 3T_5 + 4T_5 + \cdots + (n-1)T_5 + nT_5 + (n+1)T_5 \\ &= T_5 + 2T_5 + 3T_5 + 4T_5 + \cdots + (n-1)T_5 + nT_5 + (n+1)T_5 - T_5 \end{aligned}$$

which can be factored as

$$T_5\left[1 + 2 + 3 + \cdots + (n-1) + n + (n+1)\right] - T_5 = \left[\frac{1}{2}(n^2 + n)\right] T_5 + (n+1)T_5 - T_5 = T_5\left[\frac{1}{2}(n^2 + n)\right] + nT_5 = \left[\frac{1}{2}(n^2 + 3n)\right] T_5$$

Therefore the total running time for this algorithm is:

$$f(n) = T_1 + T_2 + T_3 + T_7 + (n+1)T_4 + \left[\frac{1}{2}(n^2 + n)\right] T_6 + \left[\frac{1}{2}(n^2 + 3n)\right] T_5$$

which reduces to

$$f(n) = \left[\frac{1}{2}(n^2 + n)\right] T_6 + \left[\frac{1}{2}(n^2 + 3n)\right] T_5 + (n+1)T_4 + T_1 + T_2 + T_3 + T_7$$

As a rule-of-thumb, one can assume that the highest-order term in any given function dominates its rate of growth and thus defines its run-time order. In this example, n^2 is the highest-order term, so one can conclude that f(n) =

O(n²). Formally this can be proven as follows:

Prove that $\left[\frac{1}{2}(n^2+n)\right]T_6+\left[\frac{1}{2}(n^2+3n)\right]T_5+(n+1)T_4+T_1+T_2+T_3+T_7 \le cn^2,\ n \ge n_0$

$$\left[\frac{1}{2}(n^2+n)\right]T_6+\left[\frac{1}{2}(n^2+3n)\right]T_5+(n+1)T_4+T_1+T_2+T_3+T_7$$

$$\le (n^2+n)T_6+(n^2+3n)T_5+(n+1)T_4+T_1+T_2+T_3+T_7 \text{(for } n \ge 0\text{)}$$

Let k be a constant greater than or equal to [T_1..T_7]

$$T_6(n^2+n)+T_5(n^2+3n)+(n+1)T_4+T_1+T_2+T_3+T_7 \le k(n^2+n)+k(n^2+3n)+kn+5k$$

$$= 2kn^2+5kn+5k \le 2kn^2+5kn^2+5kn^2 \text{(for } n \ge 1\text{)} = 12kn^2$$

Therefore $\left[\frac{1}{2}(n^2+n)\right]T_6+\left[\frac{1}{2}(n^2+3n)\right]T_5+(n+1)T_4+T_1+T_2+T_3+T_7 \le cn^2, n \ge n_0$

for $c = 12k, n_0 = 1$

A more elegant approach to analyzing this algorithm would be to declare that [T_1..T_7] are all equal to one unit of time, in a system of units chosen so that one unit is greater than or equal to the actual times for these steps. This would mean that the algorithm's running time breaks down as follows:[7]

$$4+\sum_{i=1}^{n} i \le 4+\sum_{i=1}^{n} n = 4+n^2 \le 5n^2 \text{(for } n \ge 1\text{)} = O(n^2).$$

Growth rate analysis of other resources

The methodology of run-time analysis can also be utilized for predicting other growth rates, such as consumption of memory space. As an example, consider the following pseudocode which manages and reallocates memory usage by a program based on the size of a file which that program manages:

```
while (file still open)
    let n = size of file
    for every 100,000 kilobytes of increase in file size
        double the amount of memory reserved
```

In this instance, as the file size n increases, memory will be consumed at an exponential growth rate, which is order O(2^n). This is an extremely rapid and most likely unmanageable growth rate for consumption of memory resources.

Relevance

Algorithm analysis is important in practice because the accidental or unintentional use of an inefficient algorithm can significantly impact system performance. In time-sensitive applications, an algorithm taking too long to run can render its results outdated or useless. An inefficient algorithm can also end up requiring an uneconomical amount of computing power or storage in order to run, again rendering it practically useless.

Notes

[1] , section 1.3

[2] Examples of the price of abstraction? (http://cstheory.stackexchange.com/questions/608/examples-of-the-price-of-abstraction), cstheory.stackexchange.com

[3] The term *lg* is often used as shorthand for $\log_2$

[4] How To Avoid O-Abuse and Bribes (http://rjlipton.wordpress.com/2009/07/24/how-to-avoid-o-abuse-and-bribes/), at the blog "Gödel's Lost Letter and P=NP" by R. J. Lipton, professor of Computer Science at Georgia Tech, recounting idea by Robert Sedgewick

[5] However, this is not the case with a quantum computer

[6] It can be proven by induction that UNIQ-math-0-af19578567ebd1f4-QINU

[7] This approach, unlike the above approach, neglects the constant time consumed by the loop tests which terminate their respective loops, but it is trivial to prove that such omission does not affect the final result

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Sequences

Array data type

Not to be confused with Array data structure.

In computer science, an **array type** is a data type that is meant to describe a collection of *elements* (values or variables), each selected by one or more indices (identifying keys) that can be computed at run time by the program. Such a collection is usually called an **array variable**, **array value**, or simply **array**.[1] By analogy with the mathematical concepts of vector and matrix, array types with one and two indices are often called **vector type** and **matrix type**, respectively.

Language support for array types may include certain built-in array data types, some syntactic constructions (*array type constructors*) that the programmer may use to define such types and declare array variables, and special notation for indexing array elements. For example, in the Pascal programming language, the declaration `type MyTable = array [1..4,1..2] of integer`, defines a new array data type called `MyTable`. The declaration `var A: MyTable` then defines a variable `A` of that type, which is an aggregate of eight elements, each being an integer variable identified by two indices. In the Pascal program, those elements are denoted `A[1,1]`, `A[1,2]`, `A[2,1]`,… `A[4,2]`.[2] Special array types are often defined by the language's standard libraries.

Arrays are distinguished from lists in that arrays allow random access, while lists only allow sequential access.Wikipedia:Citation needed Dynamic lists are also more common and easier to implement than dynamic arrays. Array types are distinguished from record types mainly because they allow the element indices to be computed at run time, as in the Pascal assignment `A[I,J] := A[N-I,2*J]`. Among other things, this feature allows a single iterative statement to process arbitrarily many elements of an array variable.

In more theoretical contexts, especially in type theory and in the description of abstract algorithms, the terms "array" and "array type" sometimes refer to an abstract data type (ADT) also called *abstract array* or may refer to an *associative array*, a mathematical model with the basic operations and behavior of a typical array type in most languages — basically, a collection of elements that are selected by indices computed at run-time.

Depending on the language, array types may overlap (or be identified with) other data types that describe aggregates of values, such as lists and strings. Array types are often implemented by array data structures, but sometimes by other means, such as hash tables, linked lists, or search trees.

History

Assembly languages and low-level languages like BCPL[3] generally have no syntactic support for arrays.

Because of the importance of array structures for efficient computation, the earliest high-level programming languages, including FORTRAN (1957), COBOL (1960), and Algol 60 (1960), provided support for multi-dimensional arrays.

Abstract arrays

An array data structure can be mathematically modeled as an abstract data structure (an *abstract array*) with two operations

get(A, I): the data stored in the element of the array A whose indices are the integer tuple I.

set(A,I,V): the array that results by setting the value of that element to V.

These operations are required to satisfy the axioms[4]

$$get(set(A,I, V), I) = V$$

$$get(set(A,I, V), J) = get(A, J) \text{ if } I \neq J$$

for any array state A, any value V, and any tuples I, J for which the operations are defined.

The first axiom means that each element behaves like a variable. The second axiom means that elements with distinct indices behave as disjoint variables, so that storing a value in one element does not affect the value of any other element.

These axioms do not place any constraints on the set of valid index tuples I, therefore this abstract model can be used for triangular matrices and other oddly-shaped arrays.

Implementations

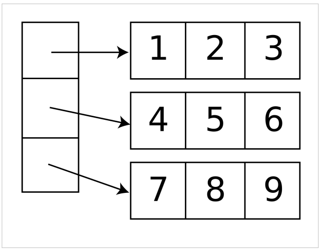
In order to effectively implement variables of such types as array structures (with indexing done by pointer arithmetic), many languages restrict the indices to integer data types (or other types that can be interpreted as integers, such as bytes and enumerated types), and require that all elements have the same data type and storage size. Most of those languages also restrict each index to a finite interval of integers, that remains fixed throughout the lifetime of the array variable. In some compiled languages, in fact, the index ranges may have to be known at compile time.

On the other hand, some programming languages provide more liberal array types, that allow indexing by arbitrary values, such as floating-point numbers, strings, objects, references, etc.. Such index values cannot be restricted to an interval, much less a fixed interval. So, these languages usually allow arbitrary new elements to be created at any time. This choice precludes the implementation of array types as array data structures. That is, those languages use array-like syntax to implement a more general associative array semantics, and must therefore be implemented by a hash table or some other search data structure.

Language support

Multi-dimensional arrays

The number of indices needed to specify an element is called the *dimension*, *dimensionality*, or rank of the array type. (This nomenclature conflicts with the concept of dimension in linear algebra,[5] where it is the number of elements. Thus, an array of numbers with 5 rows and 4 columns, hence 20 elements, is said to have dimension 2 in computing contexts, but represents a matrix with dimension 4-by-5 or 20 in mathematics. Also, the computer science meaning of "rank" is similar to its meaning in tensor algebra but not to the linear algebra concept of rank of a matrix.)



Many languages support only one-dimensional arrays. In those languages, a multi-dimensional array is typically represented by an Iliffe vector, a one-dimensional array of references to arrays of one dimension less. A two-dimensional array, in particular, would be implemented as a vector of pointers to its rows. Thus an element in row i and column j of an array A would be accessed by double indexing ($A[i][j]$ in typical notation). This way of emulating multi-dimensional arrays allows the creation of *ragged* or *jagged* arrays, where each row may have a different size — or, in general, where the valid range of each index depends on the values of all preceding indices.

This representation for multi-dimensional arrays is quite prevalent in C and C++ software. However, C and C++ will use a linear indexing formula for multi-dimensional arrays that are declared as such, e.g. by `int A[10][20]` or `int A[m][n]`, instead of the traditional `int **A`.[6]:p.81

Indexing notation

Most programming languages that support arrays support the *store* and *select* operations, and have special syntax for indexing. Early languages used parentheses, e.g. `A(i,j)`, as in FORTRAN; others choose square brackets, e.g. `A[i,j]` or `A[i][j]`, as in Algol 60 and Pascal.

Index types

Array data types are most often implemented as array structures: with the indices restricted to integer (or totally ordered) values, index ranges fixed at array creation time, and multilinear element addressing. This was the case in most "third generation" languages, and is still the case of most systems programming languages such as Ada, C, and C++. In some languages, however, array data types have the semantics of associative arrays, with indices of arbitrary type and dynamic element creation. This is the case in some scripting languages such as Awk and Lua, and of some array types provided by standard C++ libraries.

Bounds checking

Some languages (like Pascal and Modula) perform bounds checking on every access, raising an exception or aborting the program when any index is out of its valid range. Compilers may allow these checks to be turned off to trade safety for speed. Other languages (like FORTRAN and C) trust the programmer and perform no checks. Good compilers may also analyze the program to determine the range of possible values that the index may have, and this analysis may lead to bounds-checking elimination.

Index origin

Some languages, such as C, provide only zero-based array types, for which the minimum valid value for any index is 0. This choice is convenient for array implementation and address computations. With a language such as C, a pointer to the interior of any array can be defined that will symbolically act as a pseudo-array that accommodates negative indices. This works only because C does not check an index against bounds when used.

Other languages provide only *one-based* array types, where each index starts at 1; this is the traditional convention in mathematics for matrices and mathematical sequences. A few languages, such as Pascal, support *n-based* array types, whose minimum legal indices are chosen by the programmer. The relative merits of each choice have been the subject of heated debate. Zero-based indexing has a natural advantage to one-based indexing in avoiding off-by-one or fencepost errors.[7]

See comparison of programming languages (array) for the base indices used by various languages.

Highest index

The relation between numbers appearing in an array declaration and the index of that array's last element also varies by language. In many languages (such as C), one should specify the number of elements contained in the array; whereas in others (such as Pascal and Visual Basic .NET) one should specify the numeric value of the index of the last element. Needless to say, this distinction is immaterial in languages where the indices start at 1.

Array algebra

Some programming languages support array programming, where operations and functions defined for certain data types are implicitly extended to arrays of elements of those types. Thus one can write *A*+*B* to add corresponding elements of two arrays *A* and *B*. Usually these languages provide both the element-by-element multiplication and the standard matrix product of linear algebra, and which of these is represented by the * operator varies by language.

Languages providing array programming capabilities have proliferated since the innovations in this area of APL. These are core capabilities of domain-specific languages such as GAUSS, IDL, Matlab, and Mathematica. They are a core facility in newer languages, such as Julia and recent versions of Fortran. These capabilities are also provided via standard extension libraries for other general purpose programming languages (such as the widely used NumPy library for Python).

String types and arrays

Many languages provide a built-in string data type, with specialized notation ("string literals") to build values of that type. In some languages (such as C), a string is just an array of characters, or is handled in much the same way. Other languages, like Pascal, may provide vastly different operations for strings and arrays.

Array index range queries

Some programming languages provide operations that return the size (number of elements) of a vector, or, more generally, range of each index of an array. In C and C++ arrays do not support the *size* function, so programmers often have to declare separate variable to hold the size, and pass it to procedures as a separate parameter.

Elements of a newly created array may have undefined values (as in C), or may be defined to have a specific "default" value such as 0 or a null pointer (as in Java).

In C++ a std::vector object supports the *store*, *select*, and *append* operations with the performance characteristics discussed above. Vectors can be queried for their size and can be resized. Slower operations like inserting an element in the middle are also supported.

Slicing

An array slicing operation takes a subset of the elements of an array-typed entity (value or variable) and then assembles them as another array-typed entity, possibly with other indices. If array types are implemented as array structures, many useful slicing operations (such as selecting a sub-array, swapping indices, or reversing the direction of the indices) can be performed very efficiently by manipulating the dope vector of the structure. The possible slicings depend on the implementation details: for example, FORTRAN allows slicing off one column of a matrix variable, but not a row, and treat it as a vector; whereas C allow slicing off a row from a matrix, but not a column.

On the other hand, other slicing operations are possible when array types are implemented in other ways.

Resizing

Some languages allow *dynamic arrays* (also called *resizable*, *growable*, or *extensible*): array variables whose index ranges may be expanded at any time after creation, without changing the values of its current elements.

For one-dimensional arrays, this facility may be provided as an operation "`append`(*A*,*x*)" that increases the size of the array *A* by one and then sets the value of the last element to *x*. Other array types (such as Pascal strings) provide a concatenation operator, which can be used together with slicing to achieve that effect and more. In some languages, assigning a value to an element of an array automatically extends the array, if necessary, to include that element. In other array types, a slice can be replaced by an array of different size" with subsequent elements being renumbered accordingly — as in Python's list assignment "*A*[5:5] = [10,20,30]", that inserts three new elements (10,20, and 30) before element "*A*[5]". Resizable arrays are conceptually similar to lists, and the two concepts are synonymous in some languages.

An extensible array can be implemented as a fixed-size array, with a counter that records how many elements are actually in use. The `append` operation merely increments the counter; until the whole array is used, when the `append` operation may be defined to fail. This is an implementation of a dynamic array with a fixed capacity, as in the `string` type of Pascal. Alternatively, the `append` operation may re-allocate the underlying array with a larger size, and copy the old elements to the new area.

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External links

Wikibooks has a book on the topic of: ***Data Structures/Arrays***

Look up ***array*** in Wiktionary, the free dictionary.

Wikimedia Commons has media related to ***Array data structure***.

- NIST's Dictionary of Algorithms and Data Structures: Array (http://www.nist.gov/dads/HTML/array.html)

Array data structure

Not to be confused with Array data type.

In computer science, an **array data structure** or simply an **array** is a data structure consisting of a collection of *elements* (values or variables), each identified by at least one *array index* or *key*. An array is stored so that the position of each element can be computed from its index tuple by a mathematical formula. The simplest type of data structure is a linear array, also called one-dimensional array.

For example, an array of 10 32-bit integer variables, with indices 0 through 9, may be stored as 10 words at memory addresses 2000, 2004, 2008, … 2036, so that the element with index i has the address $2000 + 4 \times i$.[1]

Because the mathematical concept of a matrix can be represented as a two-dimensional grid, two-dimensional arrays are also sometimes called matrices. In some cases the term "vector" is used in computing to refer to an array, although tuples rather than vectors are more correctly the mathematical equivalent. Arrays are often used to implement tables, especially lookup tables; the word *table* is sometimes used as a synonym of *array*.

Arrays are among the oldest and most important data structures, and are used by almost every program. They are also used to implement many other data structures, such as lists and strings. They effectively exploit the addressing logic of computers. In most modern computers and many external storage devices, the memory is a one-dimensional array of words, whose indices are their addresses. Processors, especially vector processors, are often optimized for array operations.

Arrays are useful mostly because the element indices can be computed at run time. Among other things, this feature allows a single iterative statement to process arbitrarily many elements of an array. For that reason, the elements of an array data structure are required to have the same size and should use the same data representation. The set of valid index tuples and the addresses of the elements (and hence the element addressing formula) are usually,[2] but not always, fixed while the array is in use.

The term *array* is often used to mean array data type, a kind of data type provided by most high-level programming languages that consists of a collection of values or variables that can be selected by one or more indices computed at run-time. Array types are often implemented by array structures; however, in some languages they may be implemented by hash tables, linked lists, search trees, or other data structures.

The term is also used, especially in the description of algorithms, to mean associative array or "abstract array", a theoretical computer science model (an abstract data type or ADT) intended to capture the essential properties of arrays.

History

The first digital computers used machine-language programming to set up and access array structures for data tables, vector and matrix computations, and for many other purposes. Von Neumann wrote the first array-sorting program (merge sort) in 1945, during the building of the first stored-program computer.[3]p. 159 Array indexing was originally done by self-modifying code, and later using index registers and indirect addressing. Some mainframes designed in the 1960s, such as the Burroughs B5000 and its successors, used memory segmentation to perform index-bounds checking in hardware.

Assembly languages generally have no special support for arrays, other than what the machine itself provides. The earliest high-level programming languages, including FORTRAN (1957), COBOL (1960), and ALGOL 60 (1960), had support for multi-dimensional arrays, and so has C (1972). In C++ (1983), class templates exist for multi-dimensional arrays whose dimension is fixed at runtime as well as for runtime-flexible arrays.

Applications

Arrays are used to implement mathematical vectors and matrices, as well as other kinds of rectangular tables. Many databases, small and large, consist of (or include) one-dimensional arrays whose elements are records.

Arrays are used to implement other data structures, such as heaps, hash tables, deques, queues, stacks, strings, and VLists.

One or more large arrays are sometimes used to emulate in-program dynamic memory allocation, particularly memory pool allocation. Historically, this has sometimes been the only way to allocate "dynamic memory" portably.

Arrays can be used to determine partial or complete control flow in programs, as a compact alternative to (otherwise repetitive) multiple `IF` statements. They are known in this context as control tables and are used in conjunction with a purpose built interpreter whose control flow is altered according to values contained in the array. The array may contain subroutine pointers (or relative subroutine numbers that can be acted upon by SWITCH statements) that direct the path of the execution.

Element identifier and addressing formulas

When data objects are stored in an array, individual objects are selected by an index that is usually a non-negative scalar integer. Indices are also called subscripts. An index *maps* the array value to a stored object.

There are three ways in which the elements of an array can be indexed:

- **0** (*zero-based indexing*): The first element of the array is indexed by subscript of 0.
- **1** (*one-based indexing*): The first element of the array is indexed by subscript of 1.
- **n** (*n-based indexing*): The base index of an array can be freely chosen. Usually programming languages allowing *n-based indexing* also allow negative index values and other scalar data types like enumerations, or characters may be used as an array index.

Arrays can have multiple dimensions, thus it is not uncommon to access an array using multiple indices. For example a two-dimensional array `A` with three rows and four columns might provide access to the element at the

2nd row and 4th column by the expression `A[1, 3]` (in a row major language) or `A[3, 1]` (in a column major language) in the case of a zero-based indexing system. Thus two indices are used for a two-dimensional array, three for a three-dimensional array, and *n* for an *n*-dimensional array.

The number of indices needed to specify an element is called the dimension, dimensionality, or rank of the array.

In standard arrays, each index is restricted to a certain range of consecutive integers (or consecutive values of some enumerated type), and the address of an element is computed by a "linear" formula on the indices.

One-dimensional arrays

A one-dimensional array (or single dimension array) is a type of linear array. Accessing its elements involves a single subscript which can either represent a row or column index.

As an example consider the C declaration `int anArrayName[10];`

Syntax : datatype anArrayname[sizeofArray];

In the given example the array can contain 10 elements of any value available to the `int` type. In C, the array element indices are 0-9 inclusive in this case. For example, the expressions `anArrayName[0]` and `anArrayName[9]` are the first and last elements respectively.

For a vector with linear addressing, the element with index i is located at the address $B + c \times i$, where B is a fixed *base address* and c a fixed constant, sometimes called the *address increment* or *stride*.

If the valid element indices begin at 0, the constant B is simply the address of the first element of the array. For this reason, the C programming language specifies that array indices always begin at 0; and many programmers will call that element "zeroth" rather than "first".

However, one can choose the index of the first element by an appropriate choice of the base address B. For example, if the array has five elements, indexed 1 through 5, and the base address B is replaced by $B + 30c$, then the indices of those same elements will be 31 to 35. If the numbering does not start at 0, the constant B may not be the address of any element.

Multidimensional arrays

For a two-dimensional array, the element with indices i,j would have address $B + c \cdot i + d \cdot j$, where the coefficients c and d are the *row* and *column address increments*, respectively.

More generally, in a k-dimensional array, the address of an element with indices $i_1, i_2, \ldots, i_k$ is

$$B + c_1 \cdot i_1 + c_2 \cdot i_2 + \ldots + c_k \cdot i_k.$$

For example: int a[3][2];

This means that array a has 3 rows and 2 columns, and the array is of integer type. Here we can store 6 elements they are stored linearly but starting from first row linear then continuing with second row. The above array will be stored as $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$.

This formula requires only k multiplications and k additions, for any array that can fit in memory. Moreover, if any coefficient is a fixed power of 2, the multiplication can be replaced by bit shifting.

The coefficients c_k must be chosen so that every valid index tuple maps to the address of a distinct element.

If the minimum legal value for every index is 0, then B is the address of the element whose indices are all zero. As in the one-dimensional case, the element indices may be changed by changing the base address B. Thus, if a two-dimensional array has rows and columns indexed from 1 to 10 and 1 to 20, respectively, then replacing B by $B + c_1 - - 3\ c_1$ will cause them to be renumbered from 0 through 9 and 4 through 23, respectively. Taking advantage of this feature, some languages (like FORTRAN 77) specify that array indices begin at 1, as in mathematical tradition; while other languages (like Fortran 90, Pascal and Algol) let the user choose the minimum value for each index.

Dope vectors

The addressing formula is completely defined by the dimension *d*, the base address *B*, and the increments c_1, c_2, …, c_k. It is often useful to pack these parameters into a record called the array's *descriptor* or *stride vector* or *dope vector*. The size of each element, and the minimum and maximum values allowed for each index may also be included in the dope vector. The dope vector is a complete handle for the array, and is a convenient way to pass arrays as arguments to procedures. Many useful array slicing operations (such as selecting a sub-array, swapping indices, or reversing the direction of the indices) can be performed very efficiently by manipulating the dope vector.

Compact layouts

Often the coefficients are chosen so that the elements occupy a contiguous area of memory. However, that is not necessary. Even if arrays are always created with contiguous elements, some array slicing operations may create non-contiguous sub-arrays from them.

There are two systematic compact layouts for a two-dimensional array. For example, consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

In the row-major order layout (adopted by C for statically declared arrays), the elements in each row are stored in consecutive positions and all of the elements of a row have a lower address than any of the elements of a consecutive row:

1	2	3	4	5	6	7	8	9

In column-major order (traditionally used by Fortran), the elements in each column are consecutive in memory and all of the elements of a column have a lower address than any of the elements of a consecutive column:

1	4	7	2	5	8	3	6	9

For arrays with three or more indices, "row major order" puts in consecutive positions any two elements whose index tuples differ only by one in the *last* index. "Column major order" is analogous with respect to the *first* index.

In systems which use processor cache or virtual memory, scanning an array is much faster if successive elements are stored in consecutive positions in memory, rather than sparsely scattered. Many algorithms that use multidimensional arrays will scan them in a predictable order. A programmer (or a sophisticated compiler) may use this information to choose between row- or column-major layout for each array. For example, when computing the product *A*·*B* of two matrices, it would be best to have *A* stored in row-major order, and *B* in column-major order.

Resizing

Main article: Dynamic array

Static arrays have a size that is fixed when they are created and consequently do not allow elements to be inserted or removed. However, by allocating a new array and copying the contents of the old array to it, it is possible to effectively implement a *dynamic* version of an array; see dynamic array. If this operation is done infrequently, insertions at the end of the array require only amortized constant time.

Some array data structures do not reallocate storage, but do store a count of the number of elements of the array in use, called the count or size. This effectively makes the array a dynamic array with a fixed maximum size or capacity; Pascal strings are examples of this.

Non-linear formulas

More complicated (non-linear) formulas are occasionally used. For a compact two-dimensional triangular array, for instance, the addressing formula is a polynomial of degree 2.

Efficiency

Both *store* and *select* take (deterministic worst case) constant time. Arrays take linear (O(*n*)) space in the number of elements *n* that they hold.

In an array with element size *k* and on a machine with a cache line size of B bytes, iterating through an array of *n* elements requires the minimum of ceiling(*nk*/B) cache misses, because its elements occupy contiguous memory locations. This is roughly a factor of B/*k* better than the number of cache misses needed to access *n* elements at random memory locations. As a consequence, sequential iteration over an array is noticeably faster in practice than iteration over many other data structures, a property called locality of reference (this does *not* mean however, that using a perfect hash or trivial hash within the same (local) array, will not be even faster - and achievable in constant time). Libraries provide low-level optimized facilities for copying ranges of memory (such as memcpy) which can be used to move contiguous blocks of array elements significantly faster than can be achieved through individual element access. The speedup of such optimized routines varies by array element size, architecture, and implementation.

Memory-wise, arrays are compact data structures with no per-element overhead. There may be a per-array overhead, e.g. to store index bounds, but this is language-dependent. It can also happen that elements stored in an array require *less* memory than the same elements stored in individual variables, because several array elements can be stored in a single word; such arrays are often called *packed* arrays. An extreme (but commonly used) case is the bit array, where every bit represents a single element. A single octet can thus hold up to 256 different combinations of up to 8 different conditions, in the most compact form.

Array accesses with statically predictable access patterns are a major source of data parallelism.

Comparison with other data structures

Comparison of list data structures

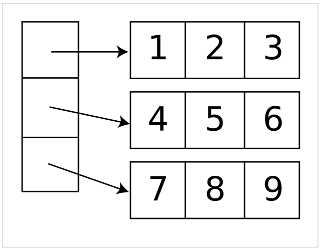
	Linked list	Array	Dynamic array	Balanced tree	Random access list
Indexing	Θ(*n*)	Θ(1)	Θ(1)	Θ(log n)	Θ(log n)
Insert/delete at beginning	Θ(1)	N/A	Θ(*n*)	Θ(log n)	Θ(1)
Insert/delete at end	Θ(*n*) when last element is unknown; Θ(1) when last element is known	N/A	Θ(1) amortized	Θ(log *n*)	Θ(log *n*) updating
Insert/delete in middle	search time + Θ(1)[4][5][6]	N/A	Θ(*n*)	Θ(log *n*)	Θ(log *n*) updating
Wasted space (average)	Θ(*n*)	0	Θ(*n*)	Θ(*n*)	Θ(*n*)

Growable arrays are similar to arrays but add the ability to insert and delete elements; adding and deleting at the end is particularly efficient. However, they reserve linear (Θ(*n*)) additional storage, whereas arrays do not reserve additional storage.

Associative arrays provide a mechanism for array-like functionality without huge storage overheads when the index values are sparse. For example, an array that contains values only at indexes 1 and 2 billion may benefit from using such a structure. Specialized associative arrays with integer keys include Patricia tries, Judy arrays, and van Emde Boas trees.

Balanced trees require O(log n) time for indexed access, but also permit inserting or deleting elements in O(log n) time,[7] whereas growable arrays require linear ($\Theta(n)$) time to insert or delete elements at an arbitrary position.

Linked lists allow constant time removal and insertion in the middle but take linear time for indexed access. Their memory use is typically worse than arrays, but is still linear.



An Iliffe vector is an alternative to a multidimensional array structure. It uses a one-dimensional array of references to arrays of one dimension less. For two dimensions, in particular, this alternative structure would be a vector of pointers to vectors, one for each row. Thus an element in row i and column j of an array A would be accessed by double indexing ($A[i][j]$ in typical notation). This alternative structure allows *ragged* or *jagged* arrays, where each row may have a different size — or, in general, where the valid range of each index depends on the values of all preceding indices. It also saves one multiplication (by the column address increment) replacing it by a bit shift (to index the vector of row pointers) and one extra memory access (fetching the row address), which may be worthwhile in some architectures.

Dimension

The dimension of an array is the number of indices needed to select an element. Thus, if the array is seen as a function on a set of possible index combinations, it is the dimension of the space of which its domain is a discrete subset. Thus a one-dimensional array is a list of data, a two-dimensional array a rectangle of data, a three-dimensional array a block of data, etc.

This should not be confused with the dimension of the set of all matrices with a given domain, that is, the number of elements in the array. For example, an array with 5 rows and 4 columns is two-dimensional, but such matrices form a 20-dimensional space. Similarly, a three-dimensional vector can be represented by a one-dimensional array of size three.

References

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[2] T. Veldhuizen. Arrays in Blitz++. In Proc. of the 2nd Int. Conf. on Scientific Computing in Object-Oriented Parallel Environments (ISCOPE), LNCS 1505, pages 223-220. Springer, 1998.
[3] Donald Knuth, *The Art of Computer Programming*, vol. 3. Addison-Wesley
[4] Gerald Kruse. CS 240 Lecture Notes (http://www.juniata.edu/faculty/kruse/cs240/syllabus.htm): Linked Lists Plus: Complexity Trade-offs (http://www.juniata.edu/faculty/kruse/cs240/linkedlist2.htm). Juniata College. Spring 2008.
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[6] *Number crunching: Why you should never, ever, EVER use linked-list in your code again* (http://kjellkod.wordpress.com/2012/02/25/why-you-should-never-ever-ever-use-linked-list-in-your-code-again/) at *kjellkod.wordpress.com*
[7] Counted B-Tree (http://www.chiark.greenend.org.uk/~sgtatham/algorithms/cbtree.html)

External links

Look up ***array*** in Wiktionary, the free dictionary.

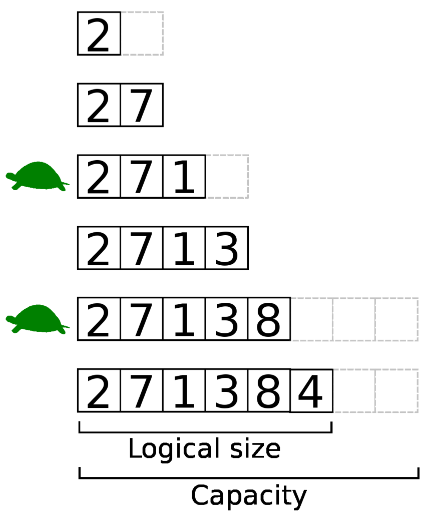
Wikibooks has a book on the topic of: ***Data Structures/Arrays***

Wikimedia Commons has media related to ***Array data structure***.

Dynamic array

In computer science, a **dynamic array**, **growable array**, **resizable array**, **dynamic table**, **mutable array**, or **array list** is a random access, variable-size list data structure that allows elements to be added or removed. It is supplied with standard libraries in many modern mainstream programming languages.

A dynamic array is not the same thing as a dynamically allocated array, which is a fixed-size array whose size is fixed when the array is allocated, although a dynamic array may use such a fixed-size array as a back end.[1]



Several values are inserted at the end of a dynamic array using geometric expansion. Grey cells indicate space reserved for expansion. Most insertions are fast (constant time), while some are slow due to the need for reallocation (Θ(*n*) time, labelled with turtles). The *logical size* and *capacity* of the final array are shown.

Bounded-size dynamic arrays and capacity

The simplest dynamic array is constructed by allocating a fixed-size array and then dividing it into two parts: the first stores the elements of the dynamic array and the second is reserved, or unused. We can then add or remove elements at the end of the dynamic array in constant time by using the reserved space, until this space is completely consumed. The number of elements used by the dynamic array contents is its *logical size* or *size*, while the size of the underlying array is called the dynamic array's *capacity* or *physical size*, which is the maximum possible size without relocating data.

In applications where the logical size is bounded, the fixed-size data structure suffices. This may be short-sighted, as more space may be needed later. A philosophical programmer may prefer to write the code to make every array capable of resizing from the outset, then return to using fixed-size arrays during program optimization. Resizing the underlying array is an expensive task, typically involving copying the entire contents of the array.

Geometric expansion and amortized cost

To avoid incurring the cost of resizing many times, dynamic arrays resize by a large amount, such as doubling in size, and use the reserved space for future expansion. The operation of adding an element to the end might work as follows:

```
function insertEnd(dynarray a, element e)
    if (a.size = a.capacity)
        // resize a to twice its current capacity:
        a.capacity ← a.capacity * 2
        // (copy the contents to the new memory location here)
    a[a.size] ← e
    a.size ← a.size + 1
```

As n elements are inserted, the capacities form a geometric progression. Expanding the array by any constant proportion ensures that inserting n elements takes $O(n)$ time overall, meaning that each insertion takes amortized constant time. The value of this proportion a leads to a time-space tradeoff: the average time per insertion operation is about $a/(a-1)$, while the number of wasted cells is bounded above by $(a-1)n$. The choice of a depends on the library or application: some textbooks use $a = 2$, but Java's ArrayList implementation uses $a = 3/2$ and the C implementation of Python's list data structure uses a = 9/8.[2]

Many dynamic arrays also deallocate some of the underlying storage if its size drops below a certain threshold, such as 30% of the capacity. This threshold must be strictly smaller than 1/a in order to support mixed sequences of insertions and removals with amortized constant cost.

Dynamic arrays are a common example when teaching amortized analysis.

Performance

Comparison of list data structures

	Linked list	Array	Dynamic array	Balanced tree	Random access list
Indexing	Θ(n)	Θ(1)	Θ(1)	Θ(log n)	Θ(log n)
Insert/delete at beginning	Θ(1)	N/A	Θ(n)	Θ(log n)	Θ(1)
Insert/delete at end	Θ(n) when last element is unknown; Θ(1) when last element is known	N/A	Θ(1) amortized	Θ(log n)	Θ(log n) updating
Insert/delete in middle	search time + Θ(1)[3][4][5]	N/A	Θ(n)	Θ(log n)	Θ(log n) updating
Wasted space (average)	Θ(n)	0	Θ(n)	Θ(n)	Θ(n)

The dynamic array has performance similar to an array, with the addition of new operations to add and remove elements:

- Getting or setting the value at a particular index (constant time)
- Iterating over the elements in order (linear time, good cache performance)
- Inserting or deleting an element in the middle of the array (linear time)
- Inserting or deleting an element at the end of the array (constant amortized time)

Dynamic arrays benefit from many of the advantages of arrays, including good locality of reference and data cache utilization, compactness (low memory use), and random access. They usually have only a small fixed additional overhead for storing information about the size and capacity. This makes dynamic arrays an attractive tool for building cache-friendly data structures. However, in languages like Python or Java that enforce reference semantics, the dynamic array generally will not store the actual data, but rather it will store references to the data that resides in other areas of memory. In this case, accessing items in the array sequentially will actually involve accessing multiple non-contiguous areas of memory, so the many advantages of the cache-friendliness of this data structure are lost.

Compared to linked lists, dynamic arrays have faster indexing (constant time versus linear time) and typically faster iteration due to improved locality of reference; however, dynamic arrays require linear time to insert or delete at an arbitrary location, since all following elements must be moved, while linked lists can do this in constant time. This disadvantage is mitigated by the gap buffer and *tiered vector* variants discussed under *Variants* below. Also, in a highly fragmented memory region, it may be expensive or impossible to find contiguous space for a large dynamic array, whereas linked lists do not require the whole data structure to be stored contiguously.

A balanced tree can store a list while providing all operations of both dynamic arrays and linked lists reasonably efficiently, but both insertion at the end and iteration over the list are slower than for a dynamic array, in theory and in practice, due to non-contiguous storage and tree traversal/manipulation overhead.

Variants

Gap buffers are similar to dynamic arrays but allow efficient insertion and deletion operations clustered near the same arbitrary location. Some deque implementations use array deques, which allow amortized constant time insertion/removal at both ends, instead of just one end.

Goodrich presented a dynamic array algorithm called *Tiered Vectors* that provided $O(n^{1/2})$ performance for order preserving insertions or deletions from the middle of the array.

Hashed Array Tree (HAT) is a dynamic array algorithm published by Sitarski in 1996. Hashed Array Tree wastes order $n^{1/2}$ amount of storage space, where n is the number of elements in the array. The algorithm has O(1) amortized performance when appending a series of objects to the end of a Hashed Array Tree.

In a 1999 paper, Brodnik et al. describe a tiered dynamic array data structure, which wastes only $n^{1/2}$ space for n elements at any point in time, and they prove a lower bound showing that any dynamic array must waste this much space if the operations are to remain amortized constant time. Additionally, they present a variant where growing and shrinking the buffer has not only amortized but worst-case constant time.

Bagwell (2002) presented the VList algorithm, which can be adapted to implement a dynamic array.

Language support

C++'s `std::vector` is an implementation of dynamic arrays, as are the `ArrayList`[6] classes supplied with the Java API and the .NET Framework. The generic `List<>` class supplied with version 2.0 of the .NET Framework is also implemented with dynamic arrays. Smalltalk's `OrderedCollection` is a dynamic array with dynamic start and end-index, making the removal of the first element also O(1). Python's `list` datatype implementation is a dynamic array. Delphi and D implement dynamic arrays at the language's core. Ada's `Ada.Containers.Vectors` generic package provides dynamic array implementation for a given subtype. Many scripting languages such as Perl and Ruby offer dynamic arrays as a built-in primitive data type. Several cross-platform frameworks provide dynamic array implementations for C: `CFArray` and `CFMutableArray` in Core Foundation; `GArray` and `GPtrArray` in GLib.

References

[1] See, for example, the source code of java.util.ArrayList class from OpenJDK 6 (http://hg.openjdk.java.net/jdk6/jdk6/jdk/file/e0e25ac28560/src/share/classes/java/util/ArrayList.java).

[2] List object implementation (http://svn.python.org/projects/python/trunk/Objects/listobject.c) from python.org, retrieved 2011-09-27.

[3] Gerald Kruse. CS 240 Lecture Notes (http://www.juniata.edu/faculty/kruse/cs240/syllabus.htm): Linked Lists Plus: Complexity Trade-offs (http://www.juniata.edu/faculty/kruse/cs240/linkedlist2.htm). Juniata College. Spring 2008.

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[5] *Number crunching: Why you should never, ever, EVER use linked-list in your code again* (http://kjellkod.wordpress.com/2012/02/25/why-you-should-never-ever-ever-use-linked-list-in-your-code-again/) at *kjellkod.wordpress.com*

[6] Javadoc on

External links

- NIST Dictionary of Algorithms and Data Structures: Dynamic array (http://www.nist.gov/dads/HTML/dynamicarray.html)
- VPOOL (http://www.bsdua.org/libbsdua.html#vpool) - C language implementation of dynamic array.
- CollectionSpy (http://www.collectionspy.com) — A Java profiler with explicit support for debugging ArrayList- and Vector-related issues.
- Open Data Structures - Chapter 2 - Array-Based Lists (http://opendatastructures.org/versions/edition-0.1e/ods-java/2_Array_Based_Lists.html)

Dictionaries

Associative array

"Dictionary (data structure)" redirects here. It is not to be confused with data dictionary.

In computer science, an **associative array**, **map**, **symbol table**, or **dictionary** is an abstract data type composed of a collection of $(key, value)$pairs, such that each possible key appears at most once in the collection.

Operations associated with this data type allow:

- the addition of pairs to the collection
- the removal of pairs from the collection
- the modification of the values of existing pairs
- the lookup of the value associated with a particular key

The **dictionary problem** is a classic computer science problem. The task of designing a data structure that maintains a set of data during 'search' 'delete' and 'insert' operations. A standard solution to the dictionary problem is a hash table; in some cases it is also possible to solve the problem using directly addressed arrays, binary search trees, or other more specialized structures.

Many programming languages include associative arrays as primitive data types, and they are available in software libraries for many others. Content-addressable memory is a form of direct hardware-level support for associative arrays.

Associative arrays have many applications including such fundamental programming patterns as memoization and the decorator pattern.[1]

Operations

In an associative array, the association between a key and a value is often known as a "binding", and the same word "binding" may also be used to refer to the process of creating a new association.

The operations that are usually defined for an associative array are:

- **Add** or **insert**: add a new $(key, value)$pair to the collection, binding the new key to its new value. The arguments to this operation are the key and the value.
- **Reassign**: replace the value in one of the $(key, value)$pairs that are already in the collection, binding an old key to a new value. As with an insertion, the arguments to this operation are the key and the value.
- **Remove** or **delete**: remove a $(key, value)$pair from the collection, unbinding a given key from its value. The argument to this operation is the key.
- **Lookup**: find the value (if any) that is bound to a given key. The argument to this operation is the key, and the value is returned from the operation. If no value is found, some associative array implementations raise an exception.

In addition, associative arrays may also include other operations such as determining the number of bindings or constructing an iterator to loop over all the bindings. Usually, for such an operation, the order in which the bindings are returned may be arbitrary.

A multimap generalizes an associative array by allowing multiple values to be associated with a single key.[2] A bidirectional map is a related abstract data type in which the bindings operate in both directions: each value must be associated with a unique key, and a second lookup operation takes a value as argument and looks up the key associated with that value.

Example

Suppose that the set of loans made by a library is to be represented in a data structure. Each book in a library may be checked out only by a single library patron at a time. However, a single patron may be able to check out multiple books. Therefore, the information about which books are checked out to which patrons may be represented by an associative array, in which the books are the keys and the patrons are the values. For instance (using notation from Python, or JSON (JavaScript Object Notation), in which a binding is represented by placing a colon between the key and the value), the current checkouts may be represented by an associative array

```
{
    "Great Expectations": "John",
    "Pride and Prejudice": "Alice",
    "Wuthering Heights": "Alice"
}
```

A lookup operation with the key "Great Expectations" in this array would return the name of the person who checked out that book, John. If John returns his book, that would cause a deletion operation in the associative array, and if Pat checks out another book, that would cause an insertion operation, leading to a different state:

```
{
    "Pride and Prejudice": "Alice",
    "The Brothers Karamazov": "Pat",
    "Wuthering Heights": "Alice"
}
```

In this new state, the same lookup as before, with the key "Great Expectations", would raise an exception, because this key is no longer present in the array.

Implementation

For dictionaries with very small numbers of bindings, it may make sense to implement the dictionary using an association list, a linked list of bindings. With this implementation, the time to perform the basic dictionary operations is linear in the total number of bindings; however, it is easy to implement and the constant factors in its running time are small.

Another very simple implementation technique, usable when the keys are restricted to a narrow range of integers, is direct addressing into an array: the value for a given key k is stored at the array cell $A[k]$, or if there is no binding for k then the cell stores a special sentinel value that indicates the absence of a binding. As well as being simple, this technique is fast: each dictionary operation takes constant time. However, the space requirement for this structure is the size of the entire keyspace, making it impractical unless the keyspace is small.

The most frequently used general purpose implementation of an associative array is with a hash table: an array of bindings, together with a hash function that maps each possible key into an array index. The basic idea of a hash table is that the binding for a given key is stored at the position given by applying the hash function to that key, and that lookup operations are performed by looking at that cell of the array and using the binding found there. However, hash table based dictionaries must be prepared to handle collisions that occur when two keys are mapped by the hash function to the same index, and many different collision resolution strategies have been developed for dealing with this situation, often based either on open addressing (looking at a sequence of hash table indices instead of a single index, until finding either the given key or an empty cell) or on hash chaining (storing a small association list instead of a single binding in each hash table cell).

Dictionaries may also be stored in binary search trees or in data structures specialized to a particular type of keys such as radix trees, tries, Judy arrays, or van Emde Boas trees, but these implementation methods are less efficient than hash tables as well as placing greater restrictions on the types of data that they can handle. The advantages of these alternative structures come from their ability to handle operations beyond the basic ones of an associative array, such as finding the binding whose key is the closest to a queried key, when the query is not itself present in the set of bindings.

Language support

Main article: Comparison of programming languages (mapping)

Associative arrays can be implemented in any programming language as a package and many language systems provide them as part of their standard library. In some languages, they are not only built into the standard system, but have special syntax, often using array-like subscripting.

Built-in syntactic support for associative arrays was introduced by SNOBOL4, under the name "table". MUMPS made multi-dimensional associative arrays, optionally persistent, its key data structure. SETL supported them as one possible implementation of sets and maps. Most modern scripting languages, starting with AWK and including Rexx, Perl, Tcl, JavaScript, Python, Ruby, and Lua, support associative arrays as a primary container type. In many more languages, they are available as library functions without special syntax.

In Smalltalk, Objective-C, .NET, Python, REALbasic, and Swift they are called *dictionaries*; in Perl, Ruby and Seed7 they are called *hashes*; in C++, Java, Go, Clojure, Scala, OCaml, Haskell they are called *maps* (see map (C++), unordered_map (C++), and `Map` [3]); in Common Lisp and Windows PowerShell, they are called *hash tables* (since both typically use this implementation). In PHP, all arrays can be associative, except that the keys are limited to integers and strings. In JavaScript (see also JSON), all objects behave as associative arrays. In Lua, they are called *tables*, and are used as the primitive building block for all data structures. In Visual FoxPro, they are called *Collections*.

References

[1] , pp. 597–599.
[2] , pp. 389–397.
[3] http://docs.oracle.com/javase/7/docs/api/java/util/Map.html

External links

Look up ***associative array*** in Wiktionary, the free dictionary.

- NIST's Dictionary of Algorithms and Data Structures: Associative Array (http://www.nist.gov/dads/HTML/assocarray.html)

Association list

Association list		
Type	associative array	
Time complexity in big O notation		
	Average	Worst case
Space	O(*n*)	O(*n*)
Search	O(*n*)	O(*n*)
Insert	O(1)	O(1)
Delete	O(*n*)	O(*n*)

In computer programming and particularly in Lisp, an **association list**, often referred to as an **alist**, is a linked list in which each list element (or node) comprises a key and a value. The association list is said to *associate* the value with the key. In order to find the value associated with a given key, each element of the list is searched in turn, starting at the head, until the key is found. Duplicate keys that appear later in the list are ignored. It is a simple way of implementing an associative array.

The disadvantage of association lists is that the time to search is O(n), where n is the length of the list. And unless the list is regularly pruned to remove elements with duplicate keys multiple values associated with the same key will increase the size of the list, and thus the time to search, without providing any compensatory advantage. One advantage is that a new element can be added to the list at its head, which can be done in constant time. For quite small values of n it is more efficient in terms of time and space than more sophisticated strategies such as hash tables and trees.

In the early development of Lisp, association lists were used to resolve references to free variables in procedures.

Many programming languages, including Lisp, Scheme, OCaml, and Haskell have functions for handling association lists in their standard library.

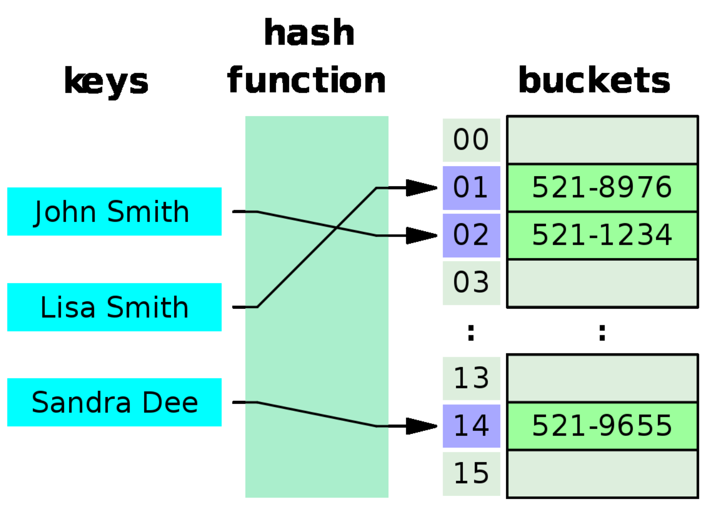
References

Hash table

Not to be confused with Hash list or Hash tree.

Hash table		
Type	Unordered associative array	
Invented	1953	
Time complexity in big O notation		
	Average	Worst case
Space	O(*n*)	O(*n*)
Search	O(1)	O(*n*)
Insert	O(1)	O(*n*)
Delete	O(1)	O(*n*)

In computing, a **hash table** (also **hash map**) is a data structure used to implement an associative array, a structure that can map keys to values. A hash table uses a hash function to compute an *index* into an array of *buckets* or *slots*, from which the correct value can be found.



A small phone book as a hash table

Ideally, the hash function will assign each key to a unique bucket, but this situation is rarely achievable in practice (usually some keys will hash to the same bucket). Instead, most hash table designs assume that *hash collisions*—different keys that are assigned by the hash function to the same bucket—will occur and must be accommodated in some way.

In a well-dimensioned hash table, the average cost (number of instructions) for each lookup is independent of the number of elements stored in the table. Many hash table designs also allow arbitrary insertions and deletions of key-value pairs, at (amortized[1]) constant average cost per operation.

In many situations, hash tables turn out to be more efficient than search trees or any other table lookup structure. For this reason, they are widely used in many kinds of computer software, particularly for associative arrays, database indexing, caches, and sets.

Hashing

Main article: Hash function

The idea of hashing is to distribute the entries (key/value pairs) across an array of *buckets*. Given a key, the algorithm computes an *index* that suggests where the entry can be found:

```
index = f(key, array_size)
```

Often this is done in two steps:

```
hash = hashfunc(key)
index = hash % array_size
```

In this method, the *hash* is independent of the array size, and it is then *reduced* to an index (a number between `0` and `array_size − 1`) using the modulo operator (`%`).

In the case that the array size is a power of two, the remainder operation is reduced to masking, which improves speed, but can increase problems with a poor hash function.

Choosing a good hash function

A good hash function and implementation algorithm are essential for good hash table performance, but may be difficult to achieve.

A basic requirement is that the function should provide a uniform distribution of hash values. A non-uniform distribution increases the number of collisions and the cost of resolving them. Uniformity is sometimes difficult to ensure by design, but may be evaluated empirically using statistical tests, e.g. a Pearson's chi-squared test for discrete uniform distributions.

The distribution needs to be uniform only for table sizes that occur in the application. In particular, if one uses dynamic resizing with exact doubling and halving of the table size *s*, then the hash function needs to be uniform only when *s* is a power of two. On the other hand, some hashing algorithms provide uniform hashes only when *s* is a prime number.[2]

For open addressing schemes, the hash function should also avoid *clustering*, the mapping of two or more keys to consecutive slots. Such clustering may cause the lookup cost to skyrocket, even if the load factor is low and collisions are infrequent. The popular multiplicative hash is claimed to have particularly poor clustering behavior.

Cryptographic hash functions are believed to provide good hash functions for any table size *s*, either by modulo reduction or by bit maskingWikipedia:Citation needed. They may also be appropriate if there is a risk of malicious users trying to sabotage a network service by submitting requests designed to generate a large number of collisions in the server's hash tables. However, the risk of sabotage can also be avoided by cheaper methods (such as applying a secret salt to the data, or using a universal hash function).

Perfect hash function

If all keys are known ahead of time, a perfect hash function can be used to create a perfect hash table that has no collisions. If minimal perfect hashing is used, every location in the hash table can be used as well.

Perfect hashing allows for constant time lookups in the worst case. This is in contrast to most chaining and open addressing methods, where the time for lookup is low on average, but may be very large (proportional to the number of entries) for some sets of keys.

Key statistics

A critical statistic for a hash table is called the *load factor*. This is simply the number of entries divided by the number of buckets, that is, *n*/*k* where *n* is the number of entries and *k* is the number of buckets.

If the load factor is kept reasonable, the hash table should perform well, provided the hashing is good. If the load factor grows too large, the hash table will become slow, or it may fail to work (depending on the method used). The expected constant time property of a hash table assumes that the load factor is kept below some bound. For a *fixed* number of buckets, the time for a lookup grows with the number of entries and so does not achieve the desired constant time.

Second to that, one can examine the variance of number of entries per bucket. For example, two tables both have 1000 entries and 1000 buckets; one has exactly one entry in each bucket, the other has all entries in the same bucket. Clearly the hashing is not working in the second one.

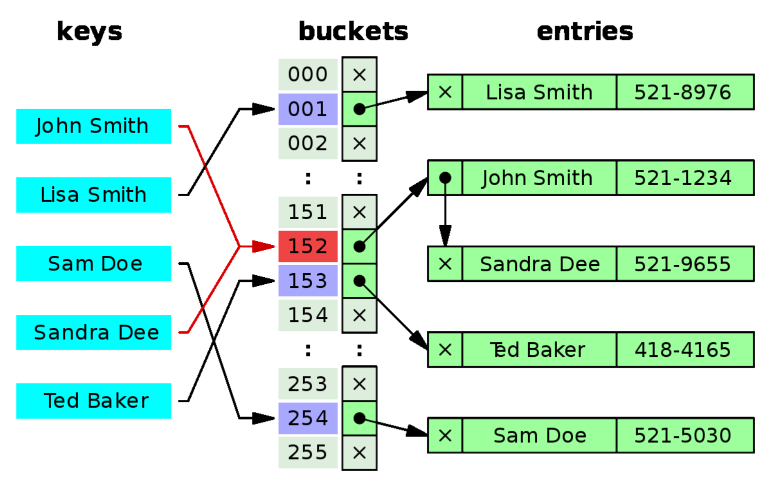
A low load factor is not especially beneficial. As the load factor approaches 0, the proportion of unused areas in the hash table increases, but there is not necessarily any reduction in search cost. This results in wasted memory.

Collision resolution

Hash collisions are practically unavoidable when hashing a random subset of a large set of possible keys. For example, if 2,450 keys are hashed into a million buckets, even with a perfectly uniform random distribution, according to the birthday problem there is approximately a 95% chance of at least two of the keys being hashed to the same slot.

Therefore, most hash table implementations have some collision resolution strategy to handle such events. Some common strategies are described below. All these methods require that the keys (or pointers to them) be stored in the table, together with the associated values.

Separate chaining



Hash collision resolved by separate chaining.

In the method known as *separate chaining*, each bucket is independent, and has some sort of list of entries with the same index. The time for hash table operations is the time to find the bucket (which is constant) plus the time for the list operation. (The technique is also called *open hashing* or *closed addressing*.)

In a good hash table, each bucket has zero or one entries, and sometimes two or three, but rarely more than that. Therefore, structures that are efficient in time and space for these cases are preferred. Structures that are efficient for a fairly large number of entries are not needed or desirable. If these cases happen often, the hashing is not working well, and this needs to be fixed.

Separate chaining with linked lists

Chained hash tables with linked lists are popular because they require only basic data structures with simple algorithms, and can use simple hash functions that are unsuitable for other methods.

The cost of a table operation is that of scanning the entries of the selected bucket for the desired key. If the distribution of keys is sufficiently uniform, the *average* cost of a lookup depends only on the average number of keys per bucket—that is, on the load factor.

Chained hash tables remain effective even when the number of table entries n is much higher than the number of slots. Their performance degrades more gracefully (linearly) with the load factor. For example, a chained hash table with 1000 slots and 10,000 stored keys (load factor 10) is five to ten times slower than a 10,000-slot table (load factor 1); but still 1000 times faster than a plain sequential list, and possibly even faster than a balanced search tree.Wikipedia:Citation needed

For separate-chaining, the worst-case scenario is when all entries are inserted into the same bucket, in which case the hash table is ineffective and the cost is that of searching the bucket data structure. If the latter is a linear list, the lookup procedure may have to scan all its entries, so the worst-case cost is proportional to the number n of entries in the table.

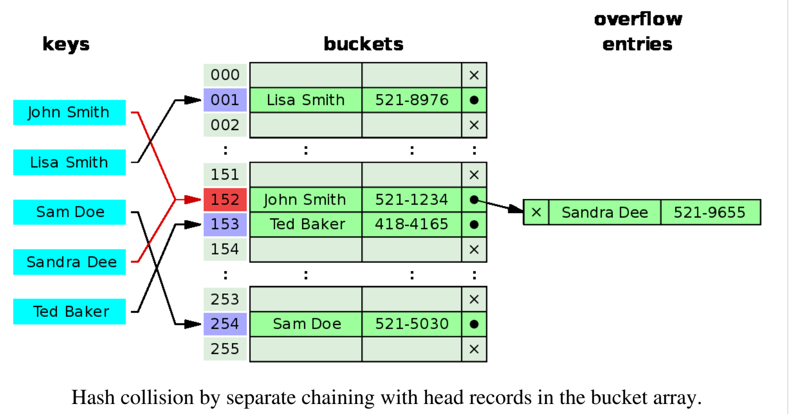
The bucket chains are often implemented as ordered lists, sorted by the key field; this choice approximately halves the average cost of unsuccessful lookups, compared to an unordered listWikipedia:Citation needed. However, if some keys are much more likely to come up than others, an unordered list with move-to-front heuristic may be more effective. More sophisticated data structures, such as balanced search trees, are worth considering only if the load factor is large (about 10 or more), or if the hash distribution is likely to be very non-uniform, or if one must guarantee good performance even in a worst-case scenario. However, using a larger table and/or a better hash function may be even more effective in those cases.

Chained hash tables also inherit the disadvantages of linked lists. When storing small keys and values, the space overhead of the `next` pointer in each entry record can be significant. An additional disadvantage is that traversing a linked list has poor cache performance, making the processor cache ineffective.

Separate chaining with list head cells

Some chaining implementations store the first record of each chain in the slot array itself. The number of pointer traversals is decreased by one for most cases. The purpose is to increase cache efficiency of hash table access.

The disadvantage is that an empty bucket takes the same space as a bucket with one entry. To save space, such hash tables often have about as many slots as stored entries, meaning that many slots have two or more entries.



Hash collision by separate chaining with head records in the bucket array.

Separate chaining with other structures

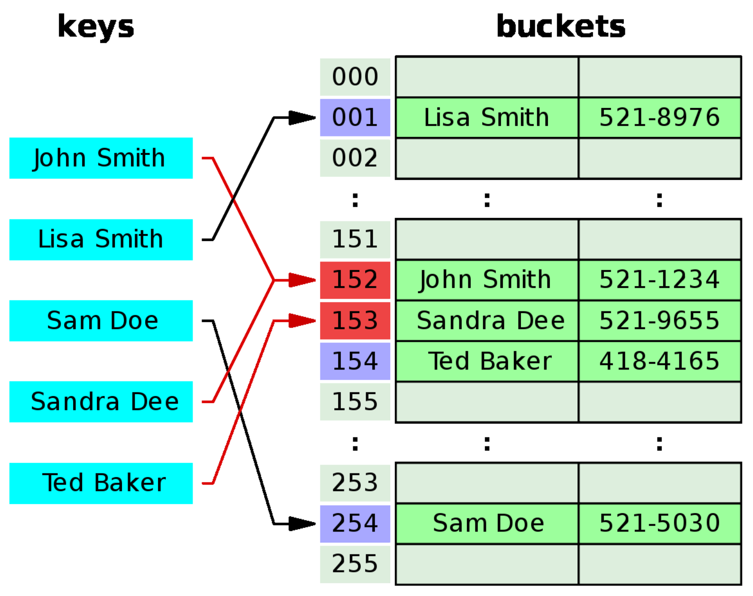
Instead of a list, one can use any other data structure that supports the required operations. For example, by using a self-balancing tree, the theoretical worst-case time of common hash table operations (insertion, deletion, lookup) can be brought down to O(log *n*) rather than O(*n*). However, this approach is only worth the trouble and extra memory cost if long delays must be avoided at all costs (e.g. in a real-time application), or if one must guard against many entries hashed to the same slot (e.g. if one expects extremely non-uniform distributions, or in the case of web sites or other publicly accessible services, which are vulnerable to malicious key distributions in requests).

The variant called array hash table uses a dynamic array to store all the entries that hash to the same slot. Each newly inserted entry gets appended to the end of the dynamic array that is assigned to the slot. The dynamic array is resized in an *exact-fit* manner, meaning it is grown only by as many bytes as needed. Alternative techniques such as growing the array by block sizes or *pages* were found to improve insertion performance, but at a cost in space. This variation makes more efficient use of CPU caching and the translation lookaside buffer (TLB), because slot entries are stored in sequential memory positions. It also dispenses with the `next` pointers that are required by linked lists, which saves space. Despite frequent array resizing, space overheads incurred by operating system such as memory fragmentation, were found to be small.

An elaboration on this approach is the so-called dynamic perfect hashing,[3] where a bucket that contains k entries is organized as a perfect hash table with k^2 slots. While it uses more memory (n^2 slots for n entries, in the worst case and $n*k$ slots in the average case), this variant has guaranteed constant worst-case lookup time, and low amortized time for insertion.

Open addressing

In another strategy, called open addressing, all entry records are stored in the bucket array itself. When a new entry has to be inserted, the buckets are examined, starting with the hashed-to slot and proceeding in some *probe sequence*, until an unoccupied slot is found. When searching for an entry, the buckets are scanned in the same sequence, until either the target record is found, or an unused array slot is found, which indicates that there is no such key in the table. The name "open addressing" refers to the fact that the location ("address") of the item is not determined by its hash value. (This method is also called **closed hashing**; it should not be confused with "open hashing" or "closed addressing" that usually mean separate chaining.)



Hash collision resolved by open addressing with linear probing (interval=1). Note that "Ted Baker" has a unique hash, but nevertheless collided with "Sandra Dee", that had previously collided with "John Smith".

Well-known probe sequences include:

- Linear probing, in which the interval between probes is fixed (usually 1)
- Quadratic probing, in which the interval between probes is increased by adding the successive outputs of a quadratic polynomial to the starting value given by the original hash computation

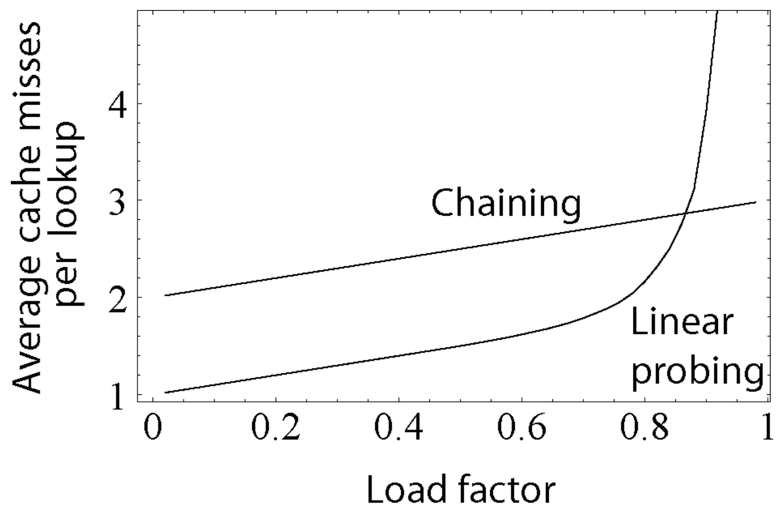
- Double hashing, in which the interval between probes is computed by another hash function

A drawback of all these open addressing schemes is that the number of stored entries cannot exceed the number of slots in the bucket array. In fact, even with good hash functions, their performance dramatically degrades when the load factor grows beyond 0.7 or so. Thus a more aggressive resize scheme is needed. Separate linking works correctly with any load factor, although performance is likely to be reasonable if it is kept below 2 or so. For many applications, these restrictions mandate the use of dynamic resizing, with its attendant costs.

Open addressing schemes also put more stringent requirements on the hash function: besides distributing the keys more uniformly over the buckets, the function must also minimize the clustering of hash values that are consecutive in the probe order. Using separate chaining, the only concern is that too many objects map to the *same* hash value; whether they are adjacent or nearby is completely irrelevant.

Open addressing only saves memory if the entries are small (less than four times the size of a pointer) and the load factor is not too small. If the load factor is close to zero (that is, there are far more buckets than stored entries), open addressing is wasteful even if each entry is just two words.

Open addressing avoids the time overhead of allocating each new entry record, and can be implemented even in the absence of a memory allocator. It also avoids the extra indirection required to access the first entry of each bucket (that is, usually the only one). It also has better locality of reference, particularly with linear probing. With small record sizes, these factors can yield better performance than chaining, particularly for lookups. Hash tables with open addressing are also easier to serialize, because they do not use pointers.



This graph compares the average number of cache misses required to look up elements in tables with chaining and linear probing. As the table passes the 80%-full mark, linear probing's performance drastically degrades.

On the other hand, normal open addressing is a poor choice for large elements, because these elements fill entire CPU cache lines (negating the cache advantage), and a large amount of space is wasted on large empty table slots. If the open addressing table only stores references to elements (external storage), it uses space comparable to chaining even for large records but loses its speed advantage.

Generally speaking, open addressing is better used for hash tables with small records that can be stored within the table (internal storage) and fit in a cache line. They are particularly suitable for elements of one word or less. If the table is expected to have a high load factor, the records are large, or the data is variable-sized, chained hash tables often perform as well or better.

Ultimately, used sensibly, any kind of hash table algorithm is usually fast *enough*; and the percentage of a calculation spent in hash table code is low. Memory usage is rarely considered excessive. Therefore, in most cases the differences between these algorithms are marginal, and other considerations typically come into play.Wikipedia:Citation needed

Coalesced hashing

A hybrid of chaining and open addressing, coalesced hashing links together chains of nodes within the table itself. Like open addressing, it achieves space usage and (somewhat diminished) cache advantages over chaining. Like chaining, it does not exhibit clustering effects; in fact, the table can be efficiently filled to a high density. Unlike chaining, it cannot have more elements than table slots.

Cuckoo hashing

Another alternative open-addressing solution is cuckoo hashing, which ensures constant lookup time in the worst case, and constant amortized time for insertions and deletions. It uses two or more hash functions, which means any key/value pair could be in two or more locations. For lookup, the first hash function is used; if the key/value is not found, then the second hash function is used, and so on. If a collision happens during insertion, then the key is re-hashed with the second hash function to map it to another bucket. If all hash functions are used and there is still a collision, then the key it collided with is removed to make space for the new key, and the old key is re-hashed with one of the other hash functions, which maps it to another bucket. If that location also results in a collision, then the process repeats until there is no collision or the process traverses all the buckets, at which point the table is resized. By combining multiple hash functions with multiple cells per bucket, very high space utilisation can be achieved.

Robin Hood hashing

One interesting variation on double-hashing collision resolution is Robin Hood hashing. The idea is that a new key may displace a key already inserted, if its probe count is larger than that of the key at the current position. The net effect of this is that it reduces worst case search times in the table. This is similar to ordered hash tables except that the criterion for bumping a key does not depend on a direct relationship between the keys. Since both the worst case and the variation in the number of probes is reduced dramatically, an interesting variation is to probe the table starting at the expected successful probe value and then expand from that position in both directions. External Robin Hashing is an extension of this algorithm where the table is stored in an external file and each table position corresponds to a fixed-sized page or bucket with *B* records.

2-choice hashing

2-choice hashing employs 2 different hash functions, $h_1(x)$ and $h_2(x)$, for the hash table. Both hash functions are used to compute two table locations. When an object is inserted in the table, then it is placed in the table location that contains fewer objects (with the default being the $h_1(x)$ table location if there is equality in bucket size). 2-choice hashing employs the principle of the power of two choices.

Hopscotch hashing

Another alternative open-addressing solution is hopscotch hashing, which combines the approaches of cuckoo hashing and linear probing, yet seems in general to avoid their limitations. In particular it works well even when the load factor grows beyond 0.9. The algorithm is well suited for implementing a resizable concurrent hash table.

The hopscotch hashing algorithm works by defining a neighborhood of buckets near the original hashed bucket, where a given entry is always found. Thus, search is limited to the number of entries in this neighborhood, which is logarithmic in the worst case, constant on average, and with proper alignment of the neighborhood typically requires one cache miss. When inserting an entry, one first attempts to add it to a bucket in the neighborhood. However, if all buckets in this neighborhood are occupied, the algorithm traverses buckets in sequence until an open slot (an unoccupied bucket) is found (as in linear probing). At that point, since the empty bucket is outside the neighborhood, items are repeatedly displaced in a sequence of hops. (This is similar to cuckoo hashing, but with the difference that in this case the empty slot is being moved into the neighborhood, instead of items being moved out with the hope of eventually finding an empty slot.) Each hop brings the open slot closer to the original neighborhood, without invalidating the neighborhood property of any of the buckets along the way. In the end, the open slot has been moved into the neighborhood, and the entry being inserted can be added to it.

Dynamic resizing

The good functioning of a hash table depends on the fact that the table size is proportional to the number of entries. With a fixed size, and the common structures, it is similar to linear search, except with a better constant factor. In some cases, the number of entries may be definitely known in advance, for example keywords in a language. More commonly, this is not known for sure, if only due to later changes in code and data. It is one serious, although common, mistake to not provide *any* way for the table to resize. A general-purpose hash table "class" will almost always have some way to resize, and it is good practice even for simple "custom" tables. An implementation should check the load factor, and do something if it becomes too large (this needs to be done only on inserts, since that is the only thing that would increase it).

To keep the load factor under a certain limit, e.g. under 3/4, many table implementations expand the table when items are inserted. For example, in Java's `HashMap` class the default load factor threshold for table expansion is 0.75 and in Python's `dict`, table size is resized when load factor is greater than 2/3.

Since buckets are usually implemented on top of a dynamic array and any constant proportion for resizing greater than 1 will keep the load factor under the desired limit, the exact choice of the constant is determined by the same space-time tradeoff as for dynamic arrays.

Resizing is accompanied by a full or incremental table **rehash** whereby existing items are mapped to new bucket locations.

To limit the proportion of memory wasted due to empty buckets, some implementations also shrink the size of the table—followed by a rehash—when items are deleted. From the point of space-time tradeoffs, this operation is similar to the deallocation in dynamic arrays.

Resizing by copying all entries

A common approach is to automatically trigger a complete resizing when the load factor exceeds some threshold $r_{\max}$. Then a new larger table is allocated, all the entries of the old table are removed and inserted into this new table, and the old table is returned to the free storage pool. Symmetrically, when the load factor falls below a second threshold $r_{\min}$, all entries are moved to a new smaller table.

If the table size increases or decreases by a fixed percentage at each expansion, the total cost of these resizings, amortized over all insert and delete operations, is still a constant, independent of the number of entries *n* and of the number *m* of operations performed.

For example, consider a table that was created with the minimum possible size and is doubled each time the load ratio exceeds some threshold. If *m* elements are inserted into that table, the total number of extra re-insertions that occur in all dynamic resizings of the table is at most $m - 1$. In other words, dynamic resizing roughly doubles the cost of each insert or delete operation.

Incremental resizing

Some hash table implementations, notably in real-time systems, cannot pay the price of enlarging the hash table all at once, because it may interrupt time-critical operations. If one cannot avoid dynamic resizing, a solution is to perform the resizing gradually:

- During the resize, allocate the new hash table, but keep the old table unchanged.
- In each lookup or delete operation, check both tables.
- Perform insertion operations only in the new table.
- At each insertion also move r elements from the old table to the new table.
- When all elements are removed from the old table, deallocate it.

To ensure that the old table is completely copied over before the new table itself needs to be enlarged, it is necessary to increase the size of the table by a factor of at least $(r + 1)/r$ during resizing.

Monotonic keys

If it is known that key values will always increase (or decrease) monotonically, then a variation of consistent hashing can be achieved by keeping a list of the single most recent key value at each hash table resize operation. Upon lookup, keys that fall in the ranges defined by these list entries are directed to the appropriate hash function—and indeed hash table—both of which can be different for each range. Since it is common to grow the overall number of entries by doubling, there will only be O(lg(N)) ranges to check, and binary search time for the redirection would be O(lg(lg(N))). As with consistent hashing, this approach guarantees that any key's hash, once issued, will never change, even when the hash table is later grown.

Other solutions

Linear hashing is a hash table algorithm that permits incremental hash table expansion. It is implemented using a single hash table, but with two possible look-up functions.

Another way to decrease the cost of table resizing is to choose a hash function in such a way that the hashes of most values do not change when the table is resized. This approach, called consistent hashing, is prevalent in disk-based and distributed hashes, where rehashing is prohibitively costly.

Performance analysis

In the simplest model, the hash function is completely unspecified and the table does not resize. For the best possible choice of hash function, a table of size k with open addressing has no collisions and holds up to k elements, with a single comparison for successful lookup, and a table of size k with chaining and n keys has the minimum $\max(0, n-k)$ collisions and $O(1 + n/k)$ comparisons for lookup. For the worst choice of hash function, every insertion causes a collision, and hash tables degenerate to linear search, with $\Omega(n)$ amortized comparisons per insertion and up to n comparisons for a successful lookup.

Adding rehashing to this model is straightforward. As in a dynamic array, geometric resizing by a factor of b implies that only n/b^i keys are inserted i or more times, so that the total number of insertions is bounded above by $bn/(b-1)$, which is $O(n)$. By using rehashing to maintain $n < k$, tables using both chaining and open addressing can have unlimited elements and perform successful lookup in a single comparison for the best choice of hash function.

In more realistic models, the hash function is a random variable over a probability distribution of hash functions, and performance is computed on average over the choice of hash function. When this distribution is uniform, the assumption is called "simple uniform hashing" and it can be shown that hashing with chaining requires $\Theta(1 + n/k)$ comparisons on average for an unsuccessful lookup, and hashing with open addressing requires $\Theta(1/(1 - n/k))$.[4] Both these bounds are constant, if we maintain $n/k < c$ using table resizing, where c is a fixed constant less than 1.

Features

Advantages

The main advantage of hash tables over other table data structures is speed. This advantage is more apparent when the number of entries is large. Hash tables are particularly efficient when the maximum number of entries can be predicted in advance, so that the bucket array can be allocated once with the optimum size and never resized.

If the set of key-value pairs is fixed and known ahead of time (so insertions and deletions are not allowed), one may reduce the average lookup cost by a careful choice of the hash function, bucket table size, and internal data structures. In particular, one may be able to devise a hash function that is collision-free, or even perfect (see below). In this case the keys need not be stored in the table.

Drawbacks

Although operations on a hash table take constant time on average, the cost of a good hash function can be significantly higher than the inner loop of the lookup algorithm for a sequential list or search tree. Thus hash tables are not effective when the number of entries is very small. (However, in some cases the high cost of computing the hash function can be mitigated by saving the hash value together with the key.)

For certain string processing applications, such as spell-checking, hash tables may be less efficient than tries, finite automata, or Judy arrays. Also, if each key is represented by a small enough number of bits, then, instead of a hash table, one may use the key directly as the index into an array of values. Note that there are no collisions in this case.

The entries stored in a hash table can be enumerated efficiently (at constant cost per entry), but only in some pseudo-random order. Therefore, there is no efficient way to locate an entry whose key is *nearest* to a given key. Listing all n entries in some specific order generally requires a separate sorting step, whose cost is proportional to $\log(n)$ per entry. In comparison, ordered search trees have lookup and insertion cost proportional to $\log(n)$, but allow finding the nearest key at about the same cost, and *ordered* enumeration of all entries at constant cost per entry.

If the keys are not stored (because the hash function is collision-free), there may be no easy way to enumerate the keys that are present in the table at any given moment.

Although the *average* cost per operation is constant and fairly small, the cost of a single operation may be quite high. In particular, if the hash table uses dynamic resizing, an insertion or deletion operation may occasionally take time proportional to the number of entries. This may be a serious drawback in real-time or interactive applications.

Hash tables in general exhibit poor locality of reference—that is, the data to be accessed is distributed seemingly at random in memory. Because hash tables cause access patterns that jump around, this can trigger microprocessor cache misses that cause long delays. Compact data structures such as arrays searched with linear search may be faster, if the table is relatively small and keys are compact. The optimal performance point varies from system to system.

Hash tables become quite inefficient when there are many collisions. While extremely uneven hash distributions are extremely unlikely to arise by chance, a malicious adversary with knowledge of the hash function may be able to supply information to a hash that creates worst-case behavior by causing excessive collisions, resulting in very poor performance, e.g. a denial of service attack.[5] In critical applications, universal hashing can be used; a data structure with better worst-case guarantees may be preferable.[6]

Uses

Associative arrays

Hash tables are commonly used to implement many types of in-memory tables. They are used to implement associative arrays (arrays whose indices are arbitrary strings or other complicated objects), especially in interpreted programming languages like AWK, Perl, and PHP.

When storing a new item into a multimap and a hash collision occurs, the multimap unconditionally stores both items.

When storing a new item into a typical associative array and a hash collision occurs, but the actual keys themselves are different, the associative array likewise stores both items. However, if the key of the new item exactly matches the key of an old item, the associative array typically erases the old item and overwrites it with the new item, so every item in the table has a unique key.

Database indexing

Hash tables may also be used as disk-based data structures and database indices (such as in dbm) although B-trees are more popular in these applications.

Caches

Hash tables can be used to implement caches, auxiliary data tables that are used to speed up the access to data that is primarily stored in slower media. In this application, hash collisions can be handled by discarding one of the two colliding entries—usually erasing the old item that is currently stored in the table and overwriting it with the new item, so every item in the table has a unique hash value.

Sets

Besides recovering the entry that has a given key, many hash table implementations can also tell whether such an entry exists or not.

Those structures can therefore be used to implement a set data structure, which merely records whether a given key belongs to a specified set of keys. In this case, the structure can be simplified by eliminating all parts that have to do with the entry values. Hashing can be used to implement both static and dynamic sets.

Object representation

Several dynamic languages, such as Perl, Python, JavaScript, and Ruby, use hash tables to implement objects. In this representation, the keys are the names of the members and methods of the object, and the values are pointers to the corresponding member or method.

Unique data representation

Hash tables can be used by some programs to avoid creating multiple character strings with the same contents. For that purpose, all strings in use by the program are stored in a single *string pool* implemented as a hash table, which is checked whenever a new string has to be created. This technique was introduced in Lisp interpreters under the name hash consing, and can be used with many other kinds of data (expression trees in a symbolic algebra system, records in a database, files in a file system, binary decision diagrams, etc.)

String interning

Main article: String interning

Implementations

In programming languages

Many programming languages provide hash table functionality, either as built-in associative arrays or as standard library modules. In C++11, for example, the `unordered_map` class provides hash tables for keys and values of arbitrary type.

In PHP 5, the Zend 2 engine uses one of the hash functions from Daniel J. Bernstein to generate the hash values used in managing the mappings of data pointers stored in a hash table. In the PHP source code, it is labelled as `DJBX33A` (Daniel J. Bernstein, Times 33 with Addition).

Python's built-in hash table implementation, in the form of the `dict` type, as well as Perl's hash type (%) are used internally to implement namespaces and therefore need to pay more attention to security, i.e. collision attacks.

In the .NET Framework, support for hash tables is provided via the non-generic `Hashtable` and generic `Dictionary` classes, which store key-value pairs, and the generic `HashSet` class, which stores only values.

Independent packages

- SparseHash [7] (formerly Google SparseHash) An extremely memory-efficient hash_map implementation, with only 2 bits/entry of overhead. The SparseHash library has several C++ hash map implementations with different performance characteristics, including one that optimizes for memory use and another that optimizes for speed.
- SunriseDD [8] An open source C library for hash table storage of arbitrary data objects with lock-free lookups, built-in reference counting and guaranteed order iteration. The library can participate in external reference counting systems or use its own built-in reference counting. It comes with a variety of hash functions and allows the use of runtime supplied hash functions via callback mechanism. Source code is well documented.
- uthash [9] This is an easy-to-use hash table for C structures.

History

The idea of hashing arose independently in different places. In January 1953, H. P. Luhn wrote an internal IBM memorandum that used hashing with chaining. G. N. Amdahl, E. M. Boehme, N. Rochester, and Arthur Samuel implemented a program using hashing at about the same time. Open addressing with linear probing (relatively prime stepping) is credited to Amdahl, but Ershov (in Russia) had the same idea.

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External links

Wikimedia Commons has media related to ***Hash tables***.

- A Hash Function for Hash Table Lookup (http://www.burtleburtle.net/bob/hash/doobs.html) by Bob Jenkins.
- Hash Tables (http://www.sparknotes.com/cs/searching/hashtables/summary.html) by SparkNotes—explanation using C
- Hash functions (http://www.azillionmonkeys.com/qed/hash.html) by Paul Hsieh
- Design of Compact and Efficient Hash Tables for Java (http://blog.griddynamics.com/2011/03/ultimate-sets-and-maps-for-java-part-i.html) link not working
- Libhashish (http://libhashish.sourceforge.net/) hash library
- NIST entry on hash tables (http://www.nist.gov/dads/HTML/hashtab.html)
- Open addressing hash table removal algorithm from ICI programming language, *ici_set_unassign* in set.c (http://ici.cvs.sourceforge.net/ici/ici/set.c?view=markup) (and other occurrences, with permission).
- A basic explanation of how the hash table works by Reliable Software (http://www.relisoft.com/book/lang/pointer/8hash.html)
- Lecture on Hash Tables (http://compgeom.cs.uiuc.edu/~jeffe/teaching/373/notes/06-hashing.pdf)
- Hash-tables in C (http://task3.cc/308/hash-maps-with-linear-probing-and-separate-chaining/)—two simple and clear examples of hash tables implementation in C with linear probing and chaining
- Open Data Structures - Chapter 5 - Hash Tables (http://opendatastructures.org/versions/edition-0.1e/ods-java/5_Hash_Tables.html)
- MIT's Introduction to Algorithms: Hashing 1 (http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-046j-introduction-to-algorithms-sma-5503-fall-2005/video-lectures/lecture-7-hashing-hash-functions/) MIT OCW lecture Video
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- How to sort a HashMap (Java) and keep the duplicate entries (http://www.lampos.net/sort-hashmap)
- How python dictionary works (http://www.laurentluce.com/posts/python-dictionary-implementation/)

Linear probing

Linear probing is a scheme in computer programming for resolving hash collisions of values of hash functions by sequentially searching the hash table for a free location.

Algorithm

Linear probing is accomplished using two values - one as a starting value and one as an interval between successive values in modular arithmetic. The second value, which is the same for all keys and known as the *stepsize*, is repeatedly added to the starting value until a free space is found, or the entire table is traversed. (In order to traverse the entire table the stepsize should be relatively prime to the arraysize, which is why the array size is often chosen to be a prime number.)

newLocation = (startingValue + stepSize) % arraySize

Given an ordinary hash function *H(x)*, a linear probing function (*H(x, i)*) would be:

$$H(x,i) = (H(x) + i) \pmod{n}.$$

Here *H(x)* is the starting value, *n* the size of the hash table, and the *stepsize* is *i* in this case.

Often, the step size is one; that is, the array cells that are probed are consecutive in the hash table. Double hashing is a variant of the same method in which the step size is itself computed by a hash function.

Properties

This algorithm, which is used in open-addressed hash tables, provides good memory caching (if stepsize is equal to one), through good locality of reference, but also results in clustering, an unfortunately high probability that where there has been one collision there will be more. The performance of linear probing is also more sensitive to input distribution when compared to double hashing, where the stepsize is determined by another hash function applied to the value instead of a fixed stepsize as in linear probing.

Dictionary operation in constant time

Using linear probing, dictionary operation can be implemented in constant time. In other words, insert, remove and find operations can be implemented in O(1), as long as the load factor of the hash table is a constant strictly less than one. This analysis makes the (unrealistic) assumption that the hash function is completely random, but can be extended also to 5-independent hash functions. Weaker properties, such as universal hashing, are not strong enough to ensure the constant-time operation of linear probing, but one practical method of hash function generation, tabulation hashing, again leads to a guaranteed constant expected time performance despite not being 5-independent.

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